Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: High School

Concepts and skills students master:

1. The complex number system includes real numbers and imaginary numbers

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Extend the properties of exponents to rational exponents. (CCSS: N-RN) i. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.1 (CCSS: N-RN.1) ii. Rewrite expressions involving radicals and rational exponents using the properties of exponents. (CCSS: N-RN.2) b. Use properties of rational and irrational numbers. (CCSS: N-RN) i. Explain why the sum or product of two rational numbers is rational. (CCSS: N-RN.3) ii. Explain why the sum of a rational number and an irrational number is irrational. (CCSS: N-RN.3) iii. Explain why the product of a nonzero rational number and an irrational number is irrational. (CCSS: N-RN.3) c. Perform arithmetic operations with complex numbers. (CCSS: N-CN) i. Define the complex number *i* such that $i2 = -1$, and show that every complex number has the form $a + bi$ where a and b are real numbers. (CCSS: N-CN.1) ii. Use the relation $i2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (CCSS: N-CN.2) d. Use complex numbers in polynomial identities and equations. (CCSS: N-CN) i. Solve quadratic equations with real coefficients that have complex solutions. (CCSS: N-CN.7)	**Inquiry Questions:** 1. When you extend to a new number systems (e.g., from integers to rational numbers and from rational numbers to real numbers), what properties apply to the extended number system? 2. Are there more complex numbers than real numbers? 3. What is a number system? 4. Why are complex numbers important? **Relevance and Application:** 1. Complex numbers have applications in fields such as chaos theory and fractals. The familiar image of the Mandelbrot fractal is the Mandelbrot set graphed on the complex plane. **Nature of Mathematics:** 1. Mathematicians build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. 2. Mathematics involves making and testing conjectures, generalizing results, and making connections among ideas, strategies, and solutions. 3. Mathematicians look for and make use of structure. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Add or subtract decimals using manipulatives/tools. II. Compute simple mathematical problems using the associative and commutative properties with whole numbers. III. Solve whole number math problems using order of operations using calculator. IV. Solve simple equations that involve multiplication and addition of one (e.g. 2 * ? + 1 = 11) where the sum is less than twenty.	**Content based access skills:** 1. Attaching meaning to mathematical functions symbols 2. Engaging in sustained participation in mathematics activities 3. Sequencing mathematical terms 4. Applying technology to solve mathematical equations

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: High School

Concepts and skills students master:

2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Reason quantitatively and use units to solve problems (CCSS: N-Q) i. Use units as a way to understand problems and to guide the solution of multi-step problems. (CCSS: N-Q.1) 1. Choose and interpret units consistently in formulas. (CCSS: N-Q.1) 2. Choose and interpret the scale and the origin in graphs and data displays. (CCSS: N-Q.1) ii. Define appropriate quantities for the purpose of descriptive modeling. (CCSS: N-Q.2) iii. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (CCSS: N-Q.3) iv. Describe factors affecting take-home pay and calculate the impact (PFL) v. Design and use a budget, including income (net take-home pay) and expenses (mortgage, car loans, and living expenses) to demonstrate how living within your means is essential for a secure financial future (PFL)	**Inquiry Questions:** 1. Can numbers ever be too big or too small to be useful? 2. How much money is enough for retirement? (PFL) 3. What is the return on investment of post-secondary educational opportunities? (PFL) **Relevance and Application:** 1. The choice of the appropriate measurement tool meets the precision requirements of the measurement task. For example, using a caliper for the manufacture of brake discs or a tape measure for pant size. 2. The reading, interpreting, and writing of numbers in scientific notation with and without technology is used extensively in the natural sciences such as representing large or small quantities such as speed of light, distance to other planets, distance between stars, the diameter of a cell, and size of a micro–organism. 3. Fluency with computation and estimation allows individuals to analyze aspects of personal finance, such as calculating a monthly budget, estimating the amount left in a checking account, making informed purchase decisions, and computing a probable paycheck given a wage (or salary), tax tables, and other deduction schedules. **Nature of Mathematics:** 1. Using mathematics to solve a problem requires choosing what mathematics to use; making simplifying assumptions, estimates, or approximations; computing; and checking to see whether the solution makes sense. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Solve real world whole number problems involving three steps or fewer labeling quantity (e.g. 23 miles, 36 dollars). (PFL)	**Content based access skills:** 1. Connecting meaning to symbols related to measurement 2. Applying technology to solve mathematical equations 3. Using materials related to mathematical problems

Standard: 1. Number Sense, Properties, and Operations
High School

[1] For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. (CCSS: N-RN.1)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Define irrational numbers.1 b. Demonstrate informally that every number has a decimal expansion. (CCSS: 8.NS.1) i. For rational numbers show that the decimal expansion repeats eventually. (CCSS: 8.NS.1) ii. Convert a decimal expansion which repeats eventually into a rational number. (CCSS: 8.NS.1) c. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.2 (CCSS: 8.NS.2) d. Apply the properties of integer exponents to generate equivalent numerical expressions.3 (CCSS: 8.EE.1) e. Use square root and cube root symbols to represent solutions to equations of the form $x2 = p$ and $x3 = p$, where p is a positive rational number. (CCSS: 8.EE.2) f. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.4 (CCSS: 8.EE.2) g. Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.5 (CCSS: 8.EE.3) h. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. (CCSS: 8.EE.4) i. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.6 (CCSS: 8.EE.4) ii. Interpret scientific notation that has been generated by technology. (CCSS: 8.EE.4)	**Inquiry Questions:** 1. Why are real numbers represented by a number line and why are the integers represented by points on the number line? 2. Why is there no real number closest to zero? 3. What is the difference between rational and irrational numbers? **Relevance and Application:** 1. Irrational numbers have applications in geometry such as the length of a diagonal of a one by one square, the height of an equilateral triangle, or the area of a circle. 2. Different representations of real numbers are used in contexts such as measurement (metric and customary units), business (profits, network down time, productivity), and community (voting rates, population density). 3. Technologies such as calculators and computers enable people to order and convert easily among fractions, decimals, and percents. **Nature of Mathematics:** 1. Mathematics provides a precise language to describe objects and events and the relationships among them. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Convert common fractions to decimals and percentages using a calculator. II. Solve multiplication problems involving powers of ten (single digit by 10 or 100).	**Content based access skills:** 1. Connecting meaning to symbols for percent and multiplication 2. Applying technology to solve mathematical equations 3. Manipulating mathematical materials and equipment

Standard: 1. Number Sense, Properties, and Operations Eighth Grade

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[1] Know that numbers that are not rational are called irrational. (CCSS: 8.NS.1)
[2] e.g., π^2. (CCSS: 8.NS.2)
For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (CCSS: 8.NS.2)
[3] For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. (CCSS: 8.EE.1)
[4] Know that $\sqrt{2}$ is irrational. (CCSS: 8.EE.2)
[5] For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger. *(CCSS: 8.EE.3)*
[6] e.g., use millimeters per year for seafloor spreading. (CCSS: 8.EE.4)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Proportional reasoning involves comparisons and multiplicative relationships among ratios

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Analyze proportional relationships and use them to solve real-world and mathematical problems.(CCSS: 7.RP) b. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.1 (CCSS: 7.RP.1) c. Identify and represent proportional relationships between quantities. (CCSS: 7.RP.2) i. Determine whether two quantities are in a proportional relationship.2 (CCSS: 7.RP.2a) ii. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (CCSS: 7.RP.2b) iii. Represent proportional relationships by equations.3 (CCSS: 7.RP.2c) iv. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. (CCSS: 7.RP.2d) d. Use proportional relationships to solve multistep ratio and percent problems.4 (CCSS: 7.RP.3) 1. Estimate and compute unit cost of consumables (to include unit conversions if necessary) sold in quantity to make purchase decisions based on cost and practicality (PFL) 2. Solve problems involving percent of a number, discounts, taxes, simple interest, percent increase, and percent decrease (PFL)	**Inquiry Questions:** 1. What information can be determined from a relative comparison that cannot be determined from an absolute comparison? 2. What comparisons can be made using ratios? 3. How do you know when a proportional relationship exists? 4. How can proportion be used to argue fairness? 5. When is it better to use an absolute comparison? 6. When is it better to use a relative comparison? **Relevance and Application:** 1. The use of ratios, rates, and proportions allows sound decision-making in daily life such as determining best values when shopping, mixing cement or paint, adjusting recipes, calculating car mileage, using speed to determine travel time, or enlarging or shrinking copies. 2. Proportional reasoning is used extensively in the workplace. For example, determine dosages for medicine; develop scale models and drawings; adjusting salaries and benefits; or prepare mixtures in laboratories. 3. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures. **Nature of Mathematics:** 1. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Demonstrate that like fractional parts are relative to the whole unit (up to 8 parts) using manipulatives/tools. II. Compute unit costs of consumables with whole number answers. (PFL)	**Content based access skills:** 1. Connecting meaning to symbols related to whole and part 2. Manipulating mathematical materials and equipment 3. Following directions during mathematical activities

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Apply understandings of addition and subtraction to add and subtract rational numbers including integers. (CCSS: 7.NS.1) i. Represent addition and subtraction on a horizontal or vertical number line diagram. (CCSS: 7.NS.1) ii. Describe situations in which opposite quantities combine to make 0.5 (CCSS: 7.NS.1a) iii. Demonstrate $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. (CCSS: 7.NS.1b) iv. Show that a number and its opposite have a sum of 0 (are additive inverses). (CCSS: 7.NS.1b) v. Interpret sums of rational numbers by describing real-world contexts. (CCSS: 7.NS.1c) vi. Demonstrate subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. (CCSS: 7.NS.1c) vii. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (CCSS: 7.NS.1c) viii. Apply properties of operations as strategies to add and subtract rational numbers. (CCSS: 7.NS.1d) b. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers including integers. (CCSS: 7.NS.2) i. Apply properties of operations to multiplication of rational numbers.6 (CCSS: 7.NS.2a) ii. Interpret products of rational numbers by describing real-world contexts. (CCSS: 7.NS.2a) iii. Apply properties of operations to divide integers.7 (CCSS: 7.NS.2b) iv. Apply properties of operations as strategies to multiply and divide rational numbers. (CCSS: 7.NS.2c) v. Convert a rational number to a decimal using long division. (CCSS: 7.NS.2d) vi. Show that the decimal form of a rational number terminates in 0s or eventually repeats. (CCSS: 7.NS.2d) c. Solve real-world and mathematical problems involving the four operations with rational numbers.8 (CCSS: 7.NS.3)	**Inquiry Questions:** 1. How do operations with rational numbers compare to operations with integers? 2. How do you know if a computational strategy is sensible? 3. Is $0.\overline{9}$ equal to one? 4. How do you know whether a fraction can be represented as a repeating or terminating decimal? **Relevance and Application:** 1. The use and understanding algorithms help individuals spend money wisely. For example, compare discounts to determine best buys and compute sales tax. 2. Estimation with rational numbers enables individuals to make decisions quickly and flexibly in daily life such as estimating a total bill at a restaurant, the amount of money left on a gift card, and price markups and markdowns. 3. People use percentages to represent quantities in real-world situations such as amount and types of taxes paid, increases or decreases in population, and changes in company profits or worker wages). **Nature of Mathematics:** 1. Mathematicians see algorithms as familiar tools in a tool chest. They combine algorithms in different ways and use them flexibly to accomplish various tasks. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. II. Identify which is larger between 1/2, 1/3 and 1/4 of a whole using manipulatives/tools. III. Calculate addition and subtraction problems involving amounts of money under ten dollars (dollars and cents) (PFL) IV. Solve multiplication and division problems with single digit multipliers and divisors.	**Content based access skills:** 1. Expressing an understanding of the concept of larger and small in relation to fractions 2. Expressing understanding that bills and coins are money and have a value 3. Manipulating mathematical materials and equipment

Standard: 1. Number Sense, Properties, and Operations
Seventh Grade

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[1] For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour. (CCSS: 7.RP.1)
[2] e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (CCSS: 7.RP.2a)
[3] For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. (CCSS: 7.RP.2c)
[4] Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. (CCSS: 7.RP.3)
[5] For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. (CCSS: 7.NS.1a)
[6] Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. (CCSS: 7.NS.2a)
[7] Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. (CCSS: 7.NS.2b)
Interpret quotients of rational numbers by describing real-world contexts. (CCSS: 7.NS.2b)
[8] Computations with rational numbers extend the rules for manipulating fractions to complex fractions. (CCSS: 7.NS.3)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Quantities can be expressed and compared using ratios and rates

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.1 (CCSS: 6.RP.1) b. Apply the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.2 (CCSS: 6.RP.2) c. Use ratio and rate reasoning to solve real-world and mathematical problems.3 (CCSS: 6.RP.3) i. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. (CCSS: 6.RP.3a) ii. Use tables to compare ratios. (CCSS: 6.RP.3a) iii. Solve unit rate problems including those involving unit pricing and constant speed.4 (CCSS: 6.RP.3b) iv. Find a percent of a quantity as a rate per 100.5 (CCSS: 6.RP.3c) v. Solve problems involving finding the whole, given a part and the percent. (CCSS: 6.RP.3c) vi. Use common fractions and percents to calculate parts of whole numbers in problem situations including comparisons of savings rates at different financial institutions (PFL) vii. Express the comparison of two whole number quantities using differences, part-to-part ratios, and part-to-whole ratios in real contexts, including investing and saving (PFL) viii. Use ratio reasoning to convert measurement units.6 (CCSS: 6.RP.3d)	**Inquiry Questions:** 1. How are ratios different from fractions? 2. What is the difference between quantity and number? **Relevance and Application:** 1. Knowledge of ratios and rates allows sound decision-making in daily life such as determining best values when shopping, creating mixtures, adjusting recipes, calculating car mileage, using speed to determine travel time, or making saving and investing decisions. 2. Ratios and rates are used to solve important problems in science, business, and politics. For example developing more fuel-efficient vehicles, understanding voter registration and voter turnout in elections, or finding more cost-effective suppliers. 3. Rates and ratios are used in mechanical devices such as bicycle gears, car transmissions, and clocks. **Nature of Mathematics:** 1. Mathematicians develop simple procedures to express complex mathematical concepts. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians reason abstractly and quantitatively. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Demonstrate the concept of a ratio using the sharing model (e.g. 15 cookies among 5 students is a ratio of 3 cookies: 1 student).	**Content based access skills:** 1. Expressing an understanding that a group of mathematical objects can be divided into smaller groups 2. Connecting meaning to symbols for numbers 3. Transitioning from one mathematical activity to another

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Fluently divide multi-digit numbers using standard algorithms. (CCSS: 6.NS.2) b. Fluently add, subtract, multiply, and divide multi-digit decimals using standard algorithms for each operation. (CCSS: 6.NS.3) c. Find the greatest common factor of two whole numbers less than or equal to 100. (CCSS: 6.NS.4) d. Find the least common multiple of two whole numbers less than or equal to 12. (CCSS: 6.NS.4) e. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.7 (CCSS: 6.NS.4) f. Interpret and model quotients of fractions through the creation of story contexts.8 (CCSS: 6.NS.1) g. Compute quotients of fractions.9 (CCSS: 6.NS.1) h. Solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.10 (CCSS: 6.NS.1)	**Inquiry Questions:** 1. Why might estimation be better than an exact answer? 2. How do operations with fractions and decimals compare to operations with whole numbers? **Relevance and Application:** 1. Rational numbers are an essential component of mathematics. Understanding fractions, decimals, and percentages is the basis for probability, proportions, measurement, money, algebra, and geometry. **Nature of Mathematics:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Add fractions with like denominators (halves, thirds and fourths). II. Compute fluently with multi-digit numbers and find common factors and multiples. III. Identify and read four digit numbers. IV. Add and subtract two digit numbers (up to a sum of 99).	**Content based access skills:** 1. Attaching meaning to mathematical symbols 2. Expressing an understanding that number symbols have word names 3. Applying technology to solve mathematical equations

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

3. In the real number system, rational numbers have a unique location on the number line and in space

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Explain why positive and negative numbers are used together to describe quantities having opposite directions or values.11 (CCSS: 6.NS.5) i. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (CCSS: 6.NS.5) b. Use number line diagrams and coordinate axes to represent points on the line and in the plane with negative number coordinates.12 (CCSS: 6.NS.6) i. Describe a rational number as a point on the number line. (CCSS: 6.NS.6) ii. Use opposite signs of numbers to indicate locations on opposite sides of 0 on the number line. (CCSS: 6.NS.6a) iii. Identify that the opposite of the opposite of a number is the number itself.13 (CCSS: 6.NS.6a) iv. Explain when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (CCSS: 6.NS.6b) v. Find and position integers and other rational numbers on a horizontal or vertical number line diagram. (CCSS: 6.NS.6c) vi. Find and position pairs of integers and other rational numbers on a coordinate plane. (CCSS: 6.NS.6c) c. Order and find absolute value of rational numbers. (CCSS: 6.NS.7) i. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.14 (CCSS: 6.NS.7a) ii. Write, interpret, and explain statements of order for rational numbers in real-world contexts.15 (CCSS: 6.NS.7b) iii. Define the absolute value of a rational number as its distance from 0 on the number line and interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.16 (CCSS: 6.NS.7c) iv. Distinguish comparisons of absolute value from statements about order.17 (CCSS: 6.NS.7d) d. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane including the use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (CCSS: 6.NS.8)	**Inquiry Questions:** 1. Why are there negative numbers? 2. How do we compare and contrast numbers? 3. Are there more rational numbers than integers? **Relevance and Application:** 1. Communication and collaboration with others is more efficient and accurate using rational numbers. For example, negotiating the price of an automobile, sharing results of a scientific experiment with the public, and planning a party with friends. 2. Negative numbers can be used to represent quantities less than zero or quantities with an associated direction such as debt, elevations below sea level, low temperatures, moving backward in time, or an object slowing down **Nature of Mathematics:** 1. Mathematicians use their understanding of relationships among numbers and the rules of number systems to create models of a wide variety of situations. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Find positive and negative numbers on a number line.	**Content based access skills:** 1. Attaching meaning to symbols related to negative numbers 2. Understanding that numerals have word names

[1] For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." (CCSS: 6.RP.1)
[2] For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (CCSS: 6.RP.2)
[3] e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (CCSS: 6.RP.3)
[4] For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? (CCSS: 6.RP.3b)
[5] e.g., 30% of a quantity means 30/100 times the quantity. (CCSS: 6.RP.3c)
[6] manipulate and transform units appropriately when multiplying or dividing quantities. (CCSS: 6.RP.3d)
[7] For example, express 36 + 8 as 4 (9 + 2). (CCSS: 6.NS.4)
[8] For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (CCSS: 6.NS.1)
[9] In general, (a/b) ÷ (c/d) = ad/bc.). (CCSS: 6.NS.1)
[10] How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? (CCSS: 6.NS.1)
[11] e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). (CCSS: 6.NS.5)
[12] Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane. (CCSS: 6.NS.6)
[13] e.g., –(–3) = 3, and that 0 is its own opposite. (CCSS: 6.NS.6a)
[14] For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. (CCSS: 6.NS.7a)
[15] For example, write –3 ºC > –7 ºC to express the fact that –3 ºC is warmer than –7 ºC. (CCSS: 6.NS.7b)
[16] For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars. (CCSS: 6.NS.7c)
[17] For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars. (CCSS: 6.NS.7d)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Explain that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. (CCSS: 5.NBT.1) i. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. (CCSS: 5.NBT.2) ii. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. (CCSS: 5.NBT.2) iii. Use whole-number exponents to denote powers of 10. (CCSS: 5.NBT.2) b. Read, write, and compare decimals to thousandths. (CCSS: 5.NBT.3) i. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.1 (CCSS: 5.NBT.3a) ii. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (CCSS: 5.NBT.3b) c. Use place value understanding to round decimals to any place. (CCSS: 5.NBT.4) d. Convert like measurement units within a given measurement system. (CCSS: 5.MD) i. Convert among different-sized standard measurement units within a given measurement system.2 (CCSS: 5.MD.1) ii. Use measurement conversions in solving multi-step, real world problems. (CCSS: 5.MD.1)	**Inquiry Questions:** 1. What is the benefit of place value system? 2. What would it mean if we did not have a place value system? 3. What is the purpose of a place value system? 4. What is the purpose of zero in a place value system? **Relevance and Application:** 1. Place value is applied to represent a myriad of numbers using only ten symbols. **Nature of Mathematics:** 1. Mathematicians use numbers like writers use letters to express ideas. 2. Mathematicians look closely and make use of structure by discerning patterns. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians reason abstractly and quantitatively. (MP) 5. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Round numbers to the next highest group of 10s. II. Express a two-digit number in expanded form using place value manipulatives up to 50. III. Identify or read three digit numbers. IV. Tell time to the quarter hour using a digital or analog clock. V. Convert equivalent values of money (two nickels/dime, five nickels/quarter, five pennies/nickel, ten pennies/dime, four quarters/dollar). (PFL)	**Content based access skills:** 1. Working collaboratively with a group around mathematical concepts 2. Understanding that money has a value and can be exchanged for goods and services 3. Expressing an understanding that coins have a specific value for each coin 4. Attending and maintaining attention to time

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Fluently multiply multi-digit whole numbers using standard algorithms. (CCSS: 5.NBT.5) b. Find whole-number quotients of whole numbers.3 (CCSS: 5.NBT.6) i. Use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS: 5.NBT.6) ii. Illustrate and explain calculations by using equations, rectangular arrays, and/or area models. (CCSS: 5.NBT.6) c. Add, subtract, multiply, and divide decimals to hundredths. (CCSS: 5.NBT.7) i. Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 5.NBT.7) ii. Relate strategies to a written method and explain the reasoning used. (CCSS: 5.NBT.7) d. Write and interpret numerical expressions. (CCSS: 5.OA) i. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. (CCSS: 5.OA.1) ii. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.4 (CCSS: 5.OA.2)	**Inquiry Questions:** 1. How are mathematical operations related? 2. What makes one strategy or algorithm better than another? **Relevance and Application:** 1. Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra. 2. There are many models of multiplication and division such as the area model for tiling a floor and the repeated addition to group people for games. **Nature of Mathematics:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians develop simple procedures to express complex mathematical concepts. 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Count by 25's and 100's. II. Generate a math sentence using appropriate symbols (>, <, +, -, =). III. Solve simple one-step equations (____ +/- a constant up to 5 =) in input/output boxes (e.g. ? + 3 = triangle).	**Content based access skills:** 1. Connecting meaning to mathematical symbols (greater than, less than, add, subtract, equals) 2. Working collaboratively with a group around mathematical concepts 3. Recognizing and reproducing a pattern 4. Applying technology to solve mathematical equations

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

3. Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use equivalent fractions as a strategy to add and subtract fractions. (CCSS: 5.NF) i. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.5 (CCSS: 5.NF.2) ii. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions6 with like denominators. (CCSS: 5.NF.1) iii. Solve word problems involving addition and subtraction of fractions referring to the same whole.7 (CCSS: 5.NF.2)	**Inquiry Questions:** 1. How do operations with fractions compare to operations with whole numbers? 2. Why are there more fractions than whole numbers? 3. Is there a smallest fraction?
	Relevance and Application: 1. Computational fluency with fractions is necessary for activities in daily life such as cooking and measuring for household projects and crafts. 2. Estimation with fractions enables quick and flexible decision-making in daily life. For example, determining how many batches of a recipe can be made with given ingredients, the amount of carpeting needed for a room, or fencing required for a backyard.
	Nature of Mathematics: 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians reason abstractly and quantitatively. (MP) 4. Mathematicians look for and make use of structure. (MP)

Extended Evidence Outcomes	Extended Readiness Competencies
With appropriate supports, students can: I. Demonstrate four quarters make a whole using visuals, numerals and manipulatives.	**Content based access skills:** 1. Expressing an understanding that groups can be separated into parts 2. Attaching meaning to a mathematical symbols for fractions (whole, half, quarter) 3. Manipulating mathematical materials and equipment

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

4. The concepts of multiplication and division can be applied to multiply and divide fractions (CCSS: 5.NF)

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). (CCSS: 5.NF.3) b. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.8 (CCSS: 5.NF.3) c. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.9 In general, (a/b) × (c/d) = ac/bd. (CCSS: 5.NF.4a) d. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. (CCSS: 5.NF.4b) i. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (CCSS: 5.NF.4b) e. Interpret multiplication as scaling (resizing). (CCSS: 5.NF.5) i. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.10 (CCSS: 5.NF.5a) ii. Apply the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. (CCSS: 5.NF.5b) f. Solve real world problems involving multiplication of fractions and mixed numbers.11 (CCSS: 5.NF.6) g. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.12 (CCSS: 5.NF.7a) h. Interpret division of a whole number by a unit fraction, and compute such quotients.13 (CCSS: 5.NF.7b) i. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.14 (CCSS: 5.NF.7c)	**Inquiry Questions:** 1. Do adding and multiplying always result in an increase? Why? 2. Do subtracting and dividing always result in a decrease? Why? 3. How do operations with fractional numbers compare to operations with whole numbers? **Relevance and Application:** 1. Rational numbers are used extensively in measurement tasks such as home remodeling, clothes alteration, graphic design, and engineering. 2. Situations from daily life can be modeled using operations with fractions, decimals, and percents such as determining the quantity of paint to buy or the number of pizzas to order for a large group. 3. Rational numbers are used to represent data and probability such as getting a certain color of gumball out of a machine, the probability that a batter will hit a home run, or the percent of a mountain covered in forest. **Nature of Mathematics:** 1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians model with mathematics. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify the meaning of the "x", "/" (i.e. multiply, divide). II. Solve multiplication and division problems using manipulatives with the total number of items less than 20 (e.g., 3 sets of 6 = 18; 20 separated into 5 sets evenly).	**Content based access skills:** 1. Attaching meaning to a mathematical symbols (multiplication, division) 2. Expressing an understanding that groups can be separated into smaller parts 3. Manipulating mathematical materials and equipment

Standard: 1. Number Sense, Properties, and Operations
Fifth Grade

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[1] e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). (CCSS: 5.NBT.3a)
[2] e.g., convert 5 cm to 0.05 m. (CCSS: 5.MD.1)
[3] with up to four-digit dividends and two-digit divisors. (CCSS: 5.NBT.6)
[4] For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product. (CCSS: 5.OA.2)
[5] For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2. (CCSS: 5.NF.2)
[6] in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.). (CCSS: 5.NF.1)
[7] including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. (CCSS: 5.NF.2)
[8] e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? (CCSS: 5.NF.3)
[9] For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (CCSS: 5.NF.4a)
[10] Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number. (CCSS: 5.NF.5b)
Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number (CCSS: 5.NF.5b)
[11] e.g., by using visual fraction models or equations to represent the problem. (CCSS: 5.NF.6)
[12] For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. (CCSS: 5.NF.7a)
[13] For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. (CCSS: 5.NF.7b)
[14] e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins? (CCSS: 5.NF.7c)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Generalize place value understanding for multi-digit whole numbers (CCSS: 4.NBT) i. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. (CCSS: 4.NBT.1) ii. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. (CCSS: 4.NBT.2) iii. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (CCSS: 4.NBT.2) iv. Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.3) b. Use decimal notation to express fractions, and compare decimal fractions (CCSS: 4.NF) i. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.1 (CCSS: 4.NF.5) ii. Use decimal notation for fractions with denominators 10 or 100.2 (CCSS: 4.NF.6) iii. Compare two decimals to hundredths by reasoning about their size.3 (CCSS: 4.NF.7)	**Inquiry Questions:** 1. Why isn't there a "oneths" place in decimal fractions? 2. How can a number with greater decimal digits be less than one with fewer decimal digits? 3. Is there a decimal closest to one? Why? **Relevance and Application:** 1. Decimal place value is the basis of the monetary system and provides information about how much items cost, how much change should be returned, or the amount of savings that has accumulated. 2. Knowledge and use of place value for large numbers provides context for population, distance between cities or landmarks, and attendance at events. **Nature of Mathematics:** 1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Arrange three sets of objects from least to most (up to 20 objects). II. Identify the meaning of the ">", "<" and "=" (i.e. more, less, equal).	**Content based access skills:** 1. Connecting meaning to mathematical symbols (greater than, less than, equals, numerals) 2. Expressing an understanding the concept of "more" and "less" 3. Manipulating mathematical materials and equipment to create sets

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

2. Different models and representations can be used to compare fractional parts

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use ideas of fraction equivalence and ordering to: (CCSS: 4.NF) i. Explain equivalence of fractions using drawings and models.4 ii. Use the principle of fraction equivalence to recognize and generate equivalent fractions. (CCSS: 4.NF.1) iii. Compare two fractions with different numerators and different denominators,5 and justify the conclusions.6 (CCSS: 4.NF.2) b. Build fractions from unit fractions by applying understandings of operations on whole numbers. (CCSS: 4.NF) i. Apply previous understandings of addition and subtraction to add and subtract fractions.7 1. Compose and decompose fractions as sums and differences of fractions with the same denominator in more than one way and justify with visual models. 2. Add and subtract mixed numbers with like denominators.8 (CCSS: 4.NF.3c) 3. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.9 (CCSS: 4.NF.3d) ii. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (CCSS: 4.NF.4) 1. Express a fraction *a*/*b* as a multiple of 1/*b*.10 (CCSS: 4.NF.4a) 2. Use a visual fraction model to express a/b as a multiple of 1/b, and apply to multiplication of whole number by a fraction.11 (CCSS: 4.NF.4b) 3. Solve word problems involving multiplication of a fraction by a whole number.12 (CCSS: 4.NF.4c)	**Inquiry Questions:** 1. How can different fractions represent the same quantity? 2. How are fractions used as models? 3. Why are fractions so useful? 4. What would the world be like without fractions? **Relevance and Application:** 1. Fractions and decimals are used any time there is a need to apportion such as sharing food, cooking, making savings plans, creating art projects, timing in music, or portioning supplies. 2. Fractions are used to represent the chance that an event will occur such as randomly selecting a certain color of shirt or the probability of a certain player scoring a soccer goal. 3. Fractions are used to measure quantities between whole units such as number of meters between houses, the height of a student, or the diameter of the moon. **Nature of Mathematics:** 1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Demonstrate equivalence of 1/2 by creating various representations of 1/2 (e.g. color every other, circle 1/2 of a set, shade 1/2 of a shape, etc.) II. Demonstrate two halves make a whole using visuals, numerals and manipulatives. III. Represent money in decimal notation ($0.05, $0.10, $0.25, $0.50, $1.00). (PFL)	**Content based access skills:** 1. Accessing and using communication system to respond to mathematical problems 2. Working collaboratively with a group around mathematical concepts 3. Expressing an Understanding that money has a value 4. Manipulating mathematical materials and equipment

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical, symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

3. Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use place value understanding and properties of operations to perform multi-digit arithmetic. (CCSS: 4.NBT) i. Fluently add and subtract multi-digit whole numbers using standard algorithms. (CCSS: 4.NBT.4) ii. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. (CCSS: 4.NBT.5) iii. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS: 4.NBT.6) iv. Illustrate and explain multiplication and division calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.6) b. Use the four operations with whole numbers to solve problems. (CCSS: 4.OA) i. Interpret a multiplication equation as a comparison.13 (CCSS: 4.OA.1) ii. Represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS: 4.OA.1) iii. Multiply or divide to solve word problems involving multiplicative comparison.14 (CCSS: 4.OA.2) iv. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. (CCSS: 4.OA.3) v. Represent multistep word problems with equations using a variable to represent the unknown quantity. (CCSS: 4.OA.3) vi. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.3) vii. Using the four operations analyze the relationship between choice and opportunity cost (PFL)	**Inquiry Questions:** 1. Is it possible to make multiplication and division of large numbers easy? 2. What do remainders mean and how are they used? 3. When is the "correct" answer not the most useful answer? **Relevance and Application:** 1. Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra. **Nature of Mathematics:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians develop simple procedures to express complex mathematical concepts. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 5. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Add up to five sets of ten using place value manipulatives. II. Solve single-digit subtraction facts using manipulatives. III. Solve single-digit addition facts using manipulatives. IV. Create sets of objects (i.e., 5 sets of 4) and find total. V. Generate addition fact families when given whole number single digit components (e.g. 1, 5, 6 creates 1+5=6, 5+1=6, 6-1=5, 6-5=1).	**Content based access skills:** 1. Understanding the concept of "more" and "less" 2. Manipulating mathematical objects to create sets 3. Responding to others in reproducing and modeling mathematical tasks 4. Expressing personal preferences and choices related to patterns

[1] For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. (CCSS: 4.NF.6)
[2] For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. (CCSS: 4.NF.6)
[3] Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. (CCSS: 4.NF.7)
[4] Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. (CCSS: 4.NF.1)
[5] e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, (CCSS: 4.NF.2)
[6] e.g., by using a visual fraction model. (CCSS: 4.NF.2)
[7] Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. (CCSS: 4.NF.3)
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (CCSS: 4.NF.3a)
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* (CCSS: 4.NF.3b)
[8] e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (CCSS: 4.NF.3c)
[9] e.g., by using visual fraction models and equations to represent the problem. (CCSS: 4.NF.3d)
[10] For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4). (CCSS: 4.NF.4a)
[11] For example, 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.) (CCSS: 4.NF.4b)
[12] e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (CCSS: 4.NF.4c)
[13] e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. (CCSS: 4.OA.1)
[14] e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (CCSS: 4.OA.2)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. The whole number system describes place value relationships and forms the foundation for efficient algorithms

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use place value and properties of operations to perform multi-digit arithmetic. (CCSS: 3.NBT) i. Use place value to round whole numbers to the nearest 10 or 100. (CCSS: 3.NBT.1) ii. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 3.NBT.2) iii. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 using strategies based on place value and properties of operations. 1 (CCSS: 3.NBT.3)	**Inquiry Questions:** 1. How do patterns in our place value system assist in comparing whole numbers? 2. How might the most commonly used number system be different if humans had twenty fingers instead of ten? **Relevance and Application:** 1. Knowledge and use of place value for large numbers provides context for distance in outer space, prehistoric timelines, and ants in a colony. 2. The building and taking apart of numbers provide a deep understanding of the base 10 number system. **Nature of Mathematics:** 1. Mathematicians use numbers like writers use letters to express ideas. 2. Mathematicians look for and make use of structure. (MP) 3. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Combine objects to complete two sets of ten and some more using place value manipulatives (e.g. bundles, ten frames, unifix cubes, etc). II. Express a two-digit number in expanded form using place value manipulatives up to 30.	**Content based access skills:** 1. Expressing an understanding that objects can be grouped to make a new number of objects 2. Attaching meaning to a numerals 3. Manipulating mathematical materials and equipment

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Third Grade

Concepts and skills students master:

2. Parts of a whole can be modeled and represented in different ways

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Develop understanding of fractions as numbers. (CCSS: 3.NF) i. Describe a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; describe a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. (CCSS: 3.NF.1) ii. Describe a fraction as a number on the number line; represent fractions on a number line diagram.2 (CCSS: 3.NF.2) iii. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (CCSS: 3.NF.3) b. Identify two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (CCSS: 3.NF.3a) c. Identify and generate simple equivalent fractions. Explain3 why the fractions are equivalent.4 (CCSS: 3.NF.3b) d. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.5 (CCSS: 3.NF.3c) e. Compare two fractions with the same numerator or the same denominator by reasoning about their size. (CCSS: 3.NF.3d) f. Explain why comparisons are valid only when the two fractions refer to the same whole. (CCSS: 3.NF.3d) g. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.6 (CCSS: 3.NF.3d)	**Inquiry Questions:** 1. How many ways can a whole number be represented? 2. How can a fraction be represented in different, equivalent forms? 3. How do we show part of unit? **Relevance and Application:** 1. Fractions are used to share fairly with friends and family such as sharing an apple with a sibling, and splitting the cost of lunch. 2. Equivalent fractions demonstrate equal quantities even when they are presented differently such as knowing that 1/2 of a box of crayons is the same as 2/4, or that 2/6 of the class is the same as 1/3. **Nature of Mathematics:** 1. Mathematicians use visual models to solve problems. 1. Mathematicians make sense of problems and persevere in solving them. (MP) 2. Mathematicians reason abstractly and quantitatively. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Demonstrate that the symbol "½" represents an item divided into 2 equal pieces.	**Content based access skills:** 1. Attaching meaning to symbols for half 2. Working collaboratively with a group around mathematical concepts 3. Accessing and using communication system to respond to mathematical problems

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Third Grade

Concepts and skills students master:

3. Multiplication and division are inverse operations and can be modeled in a variety of ways

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Represent and solve problems involving multiplication and division. (CCSS: 3.OA) i. Interpret products of whole numbers.7 (CCSS: 3.OA.1) ii. Interpret whole-number quotients of whole numbers.8 (CCSS: 3.OA.2) iii. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.9 (CCSS: 3.OA.3) iv. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.10 (CCSS: 3.OA.4) v. Model strategies to achieve a personal financial goal using arithmetic operations (PFL) b. Apply properties of multiplication and the relationship between multiplication and division. (CCSS: 3.OA) i. Apply properties of operations as strategies to multiply and divide.11 (CCSS: 3.OA.5) ii. Interpret division as an unknown-factor problem.12 (CCSS: 3.OA.6) c. Multiply and divide within 100. (CCSS: 3.OA) i. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division13 or properties of operations. (CCSS: 3.OA.7) ii. Recall from memory all products of two one-digit numbers. (CCSS: 3.OA.7) d. Solve problems involving the four operations, and identify and explain patterns in arithmetic. (CCSS: 3.OA) i. Solve two-step word problems using the four operations. (CCSS: 3.OA.8) ii. Represent two-step word problems using equations with a letter standing for the unknown quantity. (CCSS: 3.OA.8) iii. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 3.OA.8) iv. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.14 (CCSS: 3.OA.9)	**Inquiry Questions:** 1. How are multiplication and division related? 2. How can you use a multiplication or division fact to find a related fact? 3. Why was multiplication invented? Why not just add? 4. Why was division invented? Why not just subtract? **Relevance and Application:** 1. Many situations in daily life can be modeled with multiplication and division such as how many tables to set up for a party, how much food to purchase for the family, or how many teams can be created. 2. Use of multiplication and division helps to make decisions about spending allowance or gifts of money such as how many weeks of saving an allowance of $5 per week to buy a soccer ball that costs $32. **Nature of Mathematics:** 1. Mathematicians often learn concepts on a smaller scale before applying them to a larger situation. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP) 4. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Skip count by 5s and 10s to 50 including use of nickels and dimes (PFL) and clocks. II. Solve problems involving adding the same single digit number up to five times. III. Divide a set of objects into equal sets with no remainders using the sharing model. IV. Add and subtract within real life one-step story problems using objects, representations, and numerals (up to 20). V. Find the missing element in an ABAB pattern. VI. Demonstrate one less object in a set up to nine using manipulatives.	**Content based access skills:** 1. Manipulating objects and materials related to mathematics 2. Expressing an understanding that money has a value 3. Expressing personal preferences and choices related to patterns

[1] e.g., 9 × 80, 5 × 60. (CCSS: 3.NBT.3)
[2] Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. (CCSS: 3.NF.2a)
Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. (CCSS: 3.NF.2b)
[3] e.g., 1/2 = 2/4, 4/6 = 2/3). (CCSS: 3.NF.3b)
[4] e.g., by using a visual fraction model.(CCSS: 3.NF.3b)
[5] Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. (CCSS: 3.NF.3c)
[6] e.g., by using a visual fraction model. (CCSS: 3.NF.3d)
[7] e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. (CCSS: 3.OA.1)
For example, describe a context in which a total number of objects can be expressed as 5 × 7. (CCSS: 3.OA.1)
[8] e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. (CCSS: 3.OA.2)
For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8. (CCSS: 3.OA.2)
[9] e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSS: 3.OA.3)
[10] For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?. (CCSS: 3.OA.4)
[11] Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.) (CCSS: 3.OA.5)
[12] For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. (CCSS: 3.OA.6)
[13] e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8. (CCSS: 3.OA.7)
[14] For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. (CCSS: 3.OA.9)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Second Grade

Concepts and skills students master:

1. The whole number system describes place value relationships through 1,000 and forms the foundation for efficient algorithms

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use place value to read, write, count, compare, and represent numbers. (CCSS: 2.NBT) i. Represent the digits of a three-digit number as hundreds, tens, and ones.1 (CCSS: 2.NBT.1) ii. Count within 1000. (CCSS: 2.NBT.2) iii. Skip-count by 5s, 10s, and 100s. (CCSS: 2.NBT.2) iv. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (CCSS: 2.NBT.3) v. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (CCSS: 2.NBT.4) b. Use place value understanding and properties of operations to add and subtract. (CCSS: 2.NBT) i. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 2.NBT.5) ii. Add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS: 2.NBT.6) iii. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.2 (CCSS: 2.NBT.7) iv. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (CCSS: 2.NBT.8) v. Explain why addition and subtraction strategies work, using place value and the properties of operations. (CCSS: 2.NBT.9)	**Inquiry Questions:** 1. How big is 1,000? 2. How does the position of a digit in a number affect its value? **Relevance and Application:** 1. The ability to read and write numbers allows communication about quantities such as the cost of items, number of students in a school, or number of people in a theatre. 2. Place value allows people to represent large quantities. For example, 725 can be thought of as 700 + 20 + 5. **Nature of Mathematics:** 1. Mathematicians use place value to represent many numbers with only ten digits. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians look for and make use of structure. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Count using a sequential order (1, 2, 3, etc) up to 20. II. Match the symbol "10" to a bundle of 10. III. Combine objects to complete a set of ten and some more using place value manipulatives (e.g. bundles, ten frames, unifix cubes, etc.) IV. Express a two-digit number in expanded form using place value manipulatives up to 19. V. Identify the meaning of the "+" sign (i.e. combine, plus, add, more), the "=" sign (i.e. same, equal), and the "-" sign (i.e. minus, take away, less).	**Content based access skills:** 1. Understanding one to one correspondence as related to objects and numbers 2. Manipulating mathematical materials 3. Working collaboratively with a group around mathematical concepts 4. Attaching meaning to mathematical symbols

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Second Grade

Concepts and skills students master:

2. Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Represent and solve problems involving addition and subtraction. (CCSS: 2.OA) i. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.3 (CCSS: 2.OA.1) ii. Apply addition and subtraction concepts to financial decision-making (PFL) b. Fluently add and subtract within 20 using mental strategies. (CCSS: 2.OA.2) c. Know from memory all sums of two one-digit numbers. (CCSS: 2.OA.2) d. Use equal groups of objects to gain foundations for multiplication. (CCSS: 2.OA) i. Determine whether a group of objects (up to 20) has an odd or even number of members.4 (CCSS: 2.OA.3) ii. Write an equation to express an even number as a sum of two equal addends. (CCSS: 2.OA.3) iii. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns and write an equation to express the total as a sum of equal addends. (CCSS: 2.OA.4)	**Inquiry Questions:** 1. What are the ways numbers can be broken apart and put back together? 2. What could be a result of not using pennies (taking them out of circulation)? **Relevance and Application:** 1. Addition is used to find the total number of objects such as total number of animals in a zoo, total number of students in first and second grade. 2. Subtraction is used to solve problems such as how many objects are left in a set after taking some away, or how much longer one line is than another. 3. The understanding of the value of a collection of coins helps to determine how many coins are used for a purchase or checking that the amount of change is correct. **Nature of Mathematics:** 1. Mathematicians use visual models to understand addition and subtraction. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians reason abstractly and quantitatively. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Match numeral to a quantity (numerals 1- 10). II. Indicate ordinal position (first, second, third, last). III. Use objects to add within real life one-step story problems up to five. IV. Extend a repeating ABCABC pattern by one element using manipulatives. V. Demonstrate the concepts of "none" and "all." VI. Demonstrate adding one more object to a set up to 12 using manipulatives. VII. Decompose one set of up to 12 objects into two sets of objects VIII. Identify a number up to 12 as even or odd.	**Content based access skills:** 1. Applying technology to solve mathematical equations 2. Identifying a mathematical sequence 3. Understanding the concept of "none" versus "one" 4. Attaching meaning to symbol for "none"

[1] e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: (CCSS: 2.NBT.1)
100 can be thought of as a bundle of ten tens — called a "hundred." (CCSS: 2.NBT.1a)
The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (CCSS: 2.NBT.1b)
[2] Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (CCSS: 2.NBT.7)
[3] e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.OA.1)
[4] e.g., by pairing objects or counting them by 2s. (CCSS: 2.OA.3)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: First Grade

Concepts and skills students master:

1. The whole number system describes place value relationships within and beyond 100 and forms the foundation for efficient algorithms

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Count to 120 (CCSS: 1.NBT.1) i. Count starting at any number less than 120. (CCSS: 1.NBT.1) ii. Within 120, read and write numerals and represent a number of objects with a written numeral. (CCSS: 1.NBT.1) b. Represent and use the digits of a two-digit number. (CCSS: 1.NBT.2) i. Represent the digits of a two-digit number as tens and ones.1 (CCSS: 1.NBT.2) ii. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. (CCSS: 1.NBT.3) iii. Compare two sets of objects, including pennies, up to at least 25 using language such as "three more or three fewer" (PFL) c. Use place value and properties of operations to add and subtract. (CCSS: 1.NBT) i. Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of ten, using concrete models or drawings, and/or the relationship between addition and subtraction. (CCSS: 1.NBT.4) ii. Identify coins and find the value of a collection of two coins (PFL) iii. Mentally find 10 more or 10 less than any two-digit number, without counting; explain the reasoning used. (CCSS: 1.NBT.5) iv. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 1.NBT.6) v. Relate addition and subtraction strategies to a written method and explain the reasoning used. (CCSS: 1.NBT.4 and 1.NBT.6)	**Inquiry Questions:** 1. Can numbers always be related to tens? 2. Why not always count by one? 3. Why was a place value system developed? 4. How does a position of a digit affect its value? 5. How big is 100? **Relevance and Application:** 1. The comparison of numbers helps to communicate and to make sense of the world. (For example, if someone has two more dollars than another, gets four more points than another, or takes out three fewer forks than needed. **Nature of Mathematics:** 1. Mathematics involves visualization and representation of ideas. 2. Numbers are used to count and order both real and imaginary objects. 3. Mathematicians reason abstractly and quantitatively. (MP) 4. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Count using a sequential order of numbers (1, 2, 3, etc) up to 12. II. Count quantities of objects up to five. III. Represent a number of objects with a written numeral 1-5 (e.g., a jig or counting mechanism). IV. Identify numerals 1 – 10. V. Associate numeral 0 with empty sets in different settings. VI. Combine two sets of objects to make a set up to ten.	**Content based access skills:** 1. Indicating an understanding of a sequentially ordered routine 2. Attaching meaning to one object 3. Using and organizing objects related to mathematics 4. Applying technology to solve mathematical equations

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:
- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: First Grade

Concepts and skills students master:

2. Number relationships can be used to solve addition and subtraction problems

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Represent and solve problems involving addition and subtraction. (CCSS: 1.OA) i. Use addition and subtraction within 20 to solve word problems.2 (CCSS: 1.OA.1) ii. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.3 (CCSS: 1.OA.2) b. Apply properties of operations and the relationship between addition and subtraction. (CCSS: 1.OA) i. Apply properties of operations as strategies to add and subtract.4 (CCSS: 1.OA.3) ii. Relate subtraction to unknown-addend problem.5 (CCSS: 1.OA.4) c. Add and subtract within 20. (CCSS: 1.OA) i. Relate counting to addition and subtraction.6 (CCSS: 1.OA.5) ii. Add and subtract within 20 using multiple strategies.7 (CCSS: 1.OA.6) iii. Demonstrate fluency for addition and subtraction within 10. (CCSS: 1.OA.6) d. Use addition and subtraction equations to show number relationships. (CCSS: 1.OA) i. Use the equal sign to demonstrate equality in number relationships.8 (CCSS: 1.OA.7) ii. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.9 (CCSS: 1.OA.8)	**Inquiry Questions:** 1. What is addition and how is it used? 2. What is subtraction and how is it used? 3. How are addition and subtraction related? **Relevance and Application:** 1. Addition and subtraction are used to model real-world situations such as computing saving or spending, finding the number of days until a special day, or determining an amount needed to earn a reward. 2. Fluency with addition and subtraction facts helps to quickly find answers to important questions. **Nature of Mathematics:** 1. Mathematicians use addition and subtraction to take numbers apart and put them back together in order to understand number relationships. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians look for and make use of structure. (MP)

Extended Evidence Outcomes	Extended Readiness Competencies
With appropriate supports, students can: I. Identify "more", "less" or the "same" with respect to two groups of objects. II. Extend a repeating ABAB pattern by one element using manipulatives. III. Identify the meaning of the "+" sign (i.e. combine, plus, add, more) and the "=" sign (i.e. same, equal). IV. Demonstrate one-to-one correspondence between sets of the same number of objects up to five. V. Add "one more" to a given amount up to 5. VI. Combine two sets of objects to make a larger set, using sets up to five.	**Content based access skills:** 1. Manipulate mathematical materials and tools 2. Making choices related to asking for more 3. Indicating and understanding cause and effect as related to the concepts of more and less

Standard: 1. Number Sense, Properties, and Operations
First Grade

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[1] 10 can be thought of as a bundle of ten ones — called a "ten." (CCSS: 1.NBT.2a)
The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: 1.NBT.2b)
The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). (CCSS: 1.NBT.2c)
[2] involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS: 1.OA.1)
[3] e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS: 1.OA.2)
[4] Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.). (CCSS: 1.OA.3)
[5] For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. (CCSS: 1.OA.4)
[6] e.g., by counting on 2 to add 2. (CCSS: 1.OA.5)
[7] Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8
= 4); and creating equivalent but easier or known sums (e.g., adding 6 +7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (CCSS: 1.OA.6)
[8] Understand the meaning of the equal sign, and determine if equations
involving addition and subtraction are true or false. *For example, which
of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.* (CCSS: 1.OA.7)
[9] For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ?– 3, 6 + 6 = ?. (CCSS: 1.OA.8)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Kindergarten

Concepts and skills students master:

1. Whole numbers can be used to name, count, represent, and order quantity

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use number names and the count sequence. (CCSS: K.CC) i. Count to 100 by ones and by tens. (CCSS: K.CC.1) ii. Count forward beginning from a given number within the known sequence.1 (CCSS: K.CC.2) iii. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20.2 (CCSS: K.CC.3) b. Count to determine the number of objects. (CCSS: K.CC) i. Apply the relationship between numbers and quantities and connect counting to cardinality.3 (CCSS: K.CC.4) ii. Count and represent objects to 20.4 (CCSS: K.CC.5) c. Compare and instantly recognize numbers. (CCSS: K.CC) i. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.5 (CCSS: K.CC.6) ii. Compare two numbers between 1 and 10 presented as written numerals. (CCSS: K.CC.7) iii. Identify small groups of objects fewer than five without counting	**Inquiry Questions:** 1. Why do we count things? 2. Is there a wrong way to count? Why? 3. How do you know when you have more or less? 4. What does it mean to be second and how is it different than two? **Relevance and Application:** 1. Counting is used constantly in everyday life such as counting plates for the dinner table, people on a team, pets in the home, or trees in a yard. 2. Numerals are used to represent quantities. 3. People use numbers to communicate with others such as two more forks for the dinner table, one less sister than my friend, or six more dollars for a new toy. **Nature of Mathematics:** 1. Mathematics involves visualization and representation of ideas. 2. Numbers are used to count and order both real and imaginary objects. 3. Mathematicians attend to precision. (MP) 4. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Count using a sequential order of numbers (1, 2, 3, etc) up to 5. II. Discriminate numerals from other printed symbols (1 - 9) III. Count objects up to a quantity of three (pennies, bears, blocks, etc). (PFL) IV. Participate in a real life one to one correspondence activity (e.g. match each student to one paper or pencil or cracker etc). V. Identify "more" between two groups (each group containing up to five items).	**Content based access skills:** 1. Using and organizing mathematical objects 2. Manipulate mathematical materials and tools

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Kindergarten

Concepts and skills students master:

2. Composing and decomposing quantity forms the foundation for addition and subtraction

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings. (CCSS: K.OA) i. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds,6 acting out situations, verbal explanations, expressions, or equations. (CCSS: K.OA.1) ii. Solve addition and subtraction word problems, and add and subtract within 10.7 (CCSS: K.OA.2) iii. Decompose numbers less than or equal to 10 into pairs in more than one way.8 (CCSS: K.OA.3) iv. For any number from 1 to 9, find the number that makes 10 when added to the given number.9 (CCSS: K.OA.4) v. Use objects including coins and drawings to model addition and subtraction problems to 10 (PFL) b. Fluently add and subtract within 5. (CCSS: K.OA.5) c. Compose and decompose numbers 11–19 to gain foundations for place value using objects and drawings.10 (CCSS: K.NBT)	**Inquiry Questions:** 1. What happens when two quantities are combined? 2. What happens when a set of objects is separated into different sets? **Relevance and Application:** 1. People combine quantities to find a total such as number of boys and girls in a classroom or coins for a purchase. 2. People use subtraction to find what is left over such as coins left after a purchase, number of toys left after giving some away. **Nature of Mathematics:** 1. Mathematicians create models of problems that reveal relationships and meaning. 2. Mathematics involves the creative use of imagination. 3. Mathematicians reason abstractly and quantitatively. (MP) 4. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Reproduce a three-step modeled action (e.g. three claps, three stomps, etc.) II. Compose and decompose numbers up to the number three (e.g. 3 is a group of 1 and a group of 2, or one group of 3).	**Content based access skills:** 1. Accessing a communication system to respond to mathematical problems 2. Responding to others in reproducing and modeling mathematical tasks 3. Manipulating mathematical materials and tools 4. Identifying mathematical groups

[1] instead of having to begin at 1. (CCSS: K.CC.2)
[2] with 0 representing a count of no objects. (CCSS: K.CC.3)
[3] When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (CCSS: K.CC.4a)
Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. (CCSS: K.CC.4b)
Understand that each successive number name refers to a quantity that is one larger. (CCSS: K.CC.4c)
[4] Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration. (CCSS: K.CC.5)
Given a number from 1–20, count out that many objects. (CCSS: K.CC.5)
[5] e.g., by using matching and counting strategies. (CCSS: K.CC.6)
[6] e.g., claps. (CCSS: K.OA.1)
[7] e.g., by using objects or drawings to represent the problem. (CCSS: K.OA.2)
[8] e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). (CCSS: K.OA.3)
[9] e.g., by using objects or drawings, and record the answer with a drawing or equation. (CCSS: K.OA.4)
[10] Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: K.NBT.1)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Preschool

Concepts and skills students master:

1. Quantities can be represented and counted

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Count and represent objects including coins to 10 (PFL) b. Match a quantity with a numeral	**Inquiry Questions:** 1. What do numbers tell us? 2. Is there a biggest number?
	Relevance and Application: 1. Counting helps people to determine how many such as how big a family is, how many pets there are, such as how many members in one's family, how many mice on the picture book page, how many counting bears in the cup. 2. People sort things to make sense of sets of things such as sorting pencils, toys, or clothes.
	Nature of Mathematics: 1. Numbers are used to count and order objects. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Represent one by repeating a modeled action	**Content based access skills:** 1. Responding to others in reproducing mathematical tasks

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: High School

Concepts and skills students master:

1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables

Evidence Outcomes

Students can:

a. Formulate the concept of a function and use function notation. (CCSS: F-IF)
   i. Explain that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range.1 (CCSS: F-IF.1)
   ii. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (CCSS: F-IF.2)
   iii. Demonstrate that sequences are functions,2 sometimes defined recursively, whose domain is a subset of the integers. (CCSS: F-IF.3)

b. Interpret functions that arise in applications in terms of the context. (CCSS: F-IF)
   i. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features3 given a verbal description of the relationship. ★ (CCSS: F-IF.4)
   ii. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.4 ★ (CCSS: F-IF.5)
   iii. Calculate and interpret the average rate of change5 of a function over a specified interval. Estimate the rate of change from a graph.★ (CCSS: F-IF.6)

c. Analyze functions using different representations. (CCSS: F-IF)
   i. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ (CCSS: F-IF.7)
   ii. Graph linear and quadratic functions and show intercepts, maxima, and minima. (CCSS: F-IF.7a)
   iii. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (CCSS: F-IF.7b)
   iv. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (CCSS: F-IF.7c)
   v. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (CCSS: F-IF.7e)
   vi. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (CCSS: F-IF.8)
      1. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (CCSS: F-IF.8a)
      2. Use the properties of exponents to interpret expressions for exponential functions.6 (CCSS: F-IF.8b)
      3. Compare properties of two functions each represented in a different way7 (algebraically, graphically, numerically in tables, or by verbal descriptions). (CCSS: F-IF.9)

d. Build a function that models a relationship between two quantities. (CCSS: F-BF)

21st Century Skills and Readiness Competencies

Inquiry Questions:

1. Why are relations and functions represented in multiple ways?
2. How can a table, graph, and function notation be used to explain how one function family is different from and/or similar to another?
3. What is an inverse?
4. How is "inverse function" most likely related to addition and subtraction being inverse operations and to multiplication and division being inverse operations?
5. How are patterns and functions similar and different?
6. How could you visualize a function with four variables, such as $x^2 + y^2 + z^2 + w^2 = 1$?
7. Why couldn't people build skyscrapers without using functions?
8. How do symbolic transformations affect an equation, inequality, or expression?

Relevance and Application:

1. Knowledge of how to interpret rate of change of a function allows investigation of rate of return and time on the value of investments. (PFL)
2. Comprehension of rate of change of a function is important preparation for the study of calculus.
3. The ability to analyze a function for the intercepts, asymptotes, domain, range, and local and global behavior provides insights into the situations modeled by the function. For example, epidemiologists could compare the rate of flu infection among people who received flu shots to the rate of flu infection among people who did not receive a flu shot to gain insight into the effectiveness of the flu shot.
4. The exploration of multiple representations of functions develops a deeper understanding of the relationship between the variables in the function.
5. The understanding of the relationship between variables in a function allows people to use functions to model relationships in the real world such as compound interest, population growth and decay, projectile motion, or payment plans.
6. Comprehension of slope, intercepts, and common forms of linear equations allows easy retrieval of information from linear models such as rate of growth or decrease, an initial charge for services, speed of an object, or the beginning balance of an account.

i. Write a function that describes a relationship between two quantities.★ (CCSS: F-BF.1)
   1. Determine an explicit expression, a recursive process, or steps for calculation from a context. (CCSS: F-BF.1a)
   2. Combine standard function types using arithmetic operations.8 (CCSS: F-BF.1b)

ii. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ (CCSS: F-BF.2)

e. Build new functions from existing functions. (CCSS: F-BF)
   i. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of *k,9* and find the value of *k* given the graphs.10 (CCSS: F-BF.3)
   ii. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
   iii. Find inverse functions.11 (CCSS: F-BF.4)

f. Extend the domain of trigonometric functions using the unit circle. (CCSS: F-TF)
   i. Use radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (CCSS: F-TF.1)
   ii. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (CCSS: F-TF.2)

**Indicates a part of the standard connected to the mathematical practice of Modeling*

7. Understanding sequences is important preparation for calculus. Sequences can be used to represent functions including e^x, e^{x^2}, $\sin x$, and $\cos x$.

Nature of Mathematics:
1. Mathematicians use multiple representations of functions to explore the properties of functions and the properties of families of functions.
2. Mathematicians model with mathematics. (MP)
3. Mathematicians use appropriate tools strategically. (MP)
4. Mathematicians look for and make use of structure. (MP)

Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Given a numerical relationship between two variables, find the value of one variable given the value of the other (e.g. X + Y = 7, if X = 5 then find the value of Y). II. Gather data related to a simple problem and graph the results using manipulatives/tools (e.g. Graph the total cost of a given number of CDs at $10 per CD). III. Solve simple real world problem using information from graphs or tables. IV. Select the appropriate graphical representation (first quadrant) given a situation involving constant rate of change.	**Content based access skills:** 1. Attaching meaning to mathematical variables 2. Engaging in sustained participation in mathematics activities 3. Applying technology to solve mathematical equations

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: High School

Concepts and skills students master:

2. Quantitative relationships in the real world can be modeled and solved using functions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Construct and compare linear, quadratic, and exponential models and solve problems. (CCSS: F-LE) i. Distinguish between situations that can be modeled with linear functions and with exponential functions. (CCSS: F-LE.1) 1. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (CCSS: F-LE.1a) 2. Identify situations in which one quantity changes at a constant rate per unit interval relative to another. (CCSS: F-LE.1b) 3. Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (CCSS: F-LE.1c) ii. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs.12 (CCSS: F-LE.2) iii. Use graphs and tables to describe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (CCSS: F-LE.3) iv. For exponential models, express as a logarithm the solution to *abct = d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. (CCSS: F-LE.4) b. Interpret expressions for function in terms of the situation they model. (CCSS: F-LE) i. Interpret the parameters in a linear or exponential function in terms of a context. (CCSS: F-LE.5) c. Model periodic phenomena with trigonometric functions. (CCSS: F-TF) i. Choose the trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★ (CCSS: F-TF.5) d. Model personal financial situations i. Analyze* the impact of interest rates on a personal financial plan (PFL) ii. Evaluate* the costs and benefits of credit (PFL) iii. Analyze various lending sources, services, and financial institutions (PFL) **Indicates a part of the standard connected to the mathematical practice of Modeling.*	**Inquiry Questions:** 1. Why do we classify functions? 2. What phenomena can be modeled with particular functions? 3. Which financial applications can be modeled with exponential functions? Linear functions? (PFL) 4. What elementary function or functions best represent a given scatter plot of two-variable data? 5. How much would today's purchase cost tomorrow? (PFL) **Relevance and Application:** 1. The understanding of the qualitative behavior of functions allows interpretation of the qualitative behavior of systems modeled by functions such as time-distance, population growth, decay, heat transfer, and temperature of the ocean versus depth. 2. The knowledge of how functions model real-world phenomena allows exploration and improved understanding of complex systems such as how population growth may affect the environment , how interest rates or inflation affect a personal budget, how stopping distance is related to reaction time and velocity, and how volume and temperature of a gas are related. 3. Biologists use polynomial curves to model the shapes of jaw bone fossils. They analyze the polynomials to find potential evolutionary relationships among the species. 4. Physicists use basic linear and quadratic functions to model the motion of projectiles. **Nature of Mathematics:** 1. Mathematicians use their knowledge of functions to create accurate models of complex systems. 2. Mathematicians use models to better understand systems and make predictions about future systemic behavior. 3. Mathematicians reason abstractly and quantitatively. (MP) 4. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 5. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Explore nonlinear functions using graphs and the real world situations they represent. II. Predict likely events given location in a real world periodic cycle (e.g. weather patterns, students in the hall during the day).	**Content based access skills:** 1. Engaging in sustained participation in mathematics activities 2. Applying technology to solve mathematical equations 3. Expressing an understanding of predictable routines

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: High School

Concepts and skills students master:

3. Expressions can be represented in multiple, equivalent forms

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Interpret the structure of expressions.(CCSS: A-SSE) i. Interpret expressions that represent a quantity in terms of its context.★ (CCSS: A-SSE.1) 1. Interpret parts of an expression, such as terms, factors, and coefficients. (CCSS: A-SSE.1a) 2. Interpret complicated expressions by viewing one or more of their parts as a single entity.13 (CCSS: A-SSE.1b) ii. Use the structure of an expression to identify ways to rewrite it.14 (CCSS: A-SSE.2) b. Write expressions in equivalent forms to solve problems. (CCSS: A-SSE) i. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ (CCSS: A-SSE.3) 1. Factor a quadratic expression to reveal the zeros of the function it defines. (CCSS: A-SSE.3a) 2. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (CCSS: A-SSE.3b) 3. Use the properties of exponents to transform expressions for exponential functions*.15* (CCSS: A-SSE.3c) ii. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems*.16*★ (CCSS: A-SSE.4) c. Perform arithmetic operations on polynomials. (CCSS: A-APR) i. Explain that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (CCSS: A-APR.1) d. Understand the relationship between zeros and factors of polynomials. (CCSS: A-APR) i. State and apply the Remainder Theorem.17 (CCSS: A-APR.2) ii. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (CCSS: A-APR.3) e. Use polynomial identities to solve problems. (CCSS: A-APR) i. Prove polynomial identities18 and use them to describe numerical relationships. (CCSS: A-APR.4) f. Rewrite rational expressions. (CCSS: A-APR) g. Rewrite simple rational expressions in different forms.19 (CCSS: A-APR.6) **Indicates a part of the standard connected to the mathematical practice of Modeling*	**Inquiry Questions:** 1. When is it appropriate to simplify expressions? 2. The ancient Greeks multiplied binomials and found the roots of quadratic equations without algebraic notation. How can this be done? **Relevance and Application:** 1. The simplification of algebraic expressions and solving equations are tools used to solve problems in science. Scientists represent relationships between variables by developing a formula and using values obtained from experimental measurements and algebraic manipulation to determine values of quantities that are difficult or impossible to measure directly such as acceleration due to gravity, speed of light, and mass of the earth. 2. The manipulation of expressions and solving formulas are techniques used to solve problems in geometry such as finding the area of a circle, determining the volume of a sphere, calculating the surface area of a prism, and applying the Pythagorean Theorem. **Nature of Mathematics:** 1. Mathematicians abstract a problem by representing it as an equation. They travel between the concrete problem and the abstraction to gain insights and find solutions. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP)

Extended Evidence Outcomes	Extended Readiness Competencies
With appropriate supports, students can: I. Demonstrate the commutative property (e.g. 3 x 5 = 5 x 3). II. Determine gross pay given hours worked rounded to the nearest quarter hour (less than 20 hours) and hourly rate expressed in dollars and cents using a calculator. (PFL) III. Explore sums and products of even and odd numbers to identify if the answer is even or odd. IV. Compute multiplication problems with zero as a factor. V. Find equivalent fractions (e.g. 2/4 = 1/2).	**Content based access skills:** 1. Attaching meaning to mathematical functions symbols 2. Applying technology to solve mathematical equations 3. Working cooperatively with others during mathematical activities 4. Manipulating mathematical materials and equipment

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: High School

Concepts and skills students master:

4. Solutions to equations, inequalities and systems of equations are found using a variety of tools

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Create equations that describe numbers or relationships. (CCSS: A-CED) i. Create equations and inequalities20 in one variable and use them to solve problems. (CCSS: A-CED.1) ii. Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. (CCSS: A-CED.2) iii. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.21 (CCSS: A-CED.3) iv. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.22 (CCSS: A-CED.4) b. Understand solving equations as a process of reasoning and explain the reasoning. (CCSS: A-REI) i. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. (CCSS: A-REI.1) ii. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (CCSS: A-REI.2) c. Solve equations and inequalities in one variable. (CCSS: A-REI) i. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (CCSS: A-REI.3) ii. Solve quadratic equations in one variable. (CCSS: A-REI.4) 1. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)2 = q$ that has the same solutions. Derive the quadratic formula from this form. (CCSS: A-REI.4a) 2. Solve quadratic equations23 by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. (CCSS: A-REI.4b) 3. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. (CCSS: A-REI.4b) d. Solve systems of equations. (CCSS: A-REI) i. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (CCSS: A-REI.5) ii. Solve systems of linear equations exactly and approximately,24 focusing on pairs of linear equations in two variables. (CCSS: A-REI.6) iii. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.25 (CCSS: A-REI.7) e. Represent and solve equations and inequalities graphically. (CCSS: A-REI) i. Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve.26 (CCSS: A-REI.10) ii. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$;27 find the solutions approximately.28★ (CCSS: A-REI.11) iii. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (CCSS: A-REI.12)	**Inquiry Questions:** 1. What are some similarities in solving all types of equations? 2. Why do different types of equations require different types of solution processes? 3. Can computers solve algebraic problems that people cannot solve? Why? 4. How are order of operations and operational relationships important when solving multivariable equations? **Relevance and Application:** 1. Linear programming allows representation of the constraints in a real-world situation identification of a feasible region and determination of the maximum or minimum value such as to optimize profit, or to minimize expense. 2. Effective use of graphing technology helps to find solutions to equations or systems of equations. **Nature of Mathematics:** 1. Mathematics involves visualization. 2. Mathematicians use tools to create visual representations of problems and ideas that reveal relationships and meaning. 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians use appropriate tools strategically. (MP)

Indicates a part of the standard connected to the mathematical practice of Modeling	
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Create a rule for a function given a table of values using addition (e.g. output = input + 3). II. Solve a simple equation with like denominators and one variable. III. Determine overage when the amount available is more than what is required. IV. Find values that satisfy a simple inequality (whole number answers less than 10). V. Explore a system of linear equations based on real world situations (e.g. How many hours must you work each month to meet your bills?) VI. Explore the graph of a linear function based on real world situations (e.g. How much money do I have if I work a given number of hours?)	**Content based access skills:** 1. Expressing an understanding that money is earned 2. Applying technology to solve mathematical equations 3. Manipulating mathematical materials and equipment

Standard: 2. Patterns, Functions, and Algebraic Structures
High School

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[1] If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).F-IF.1)
[2] For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. (CCSS: F-IF.3)
[3] Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (CCSS: F-IF.4)
[4] For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (CCSS: F-IF.5)
[5] presented symbolically or as a table. (CCSS: F-IF.6)
[6] For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10,. (CCSS: F-IF.8b)
[7] For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (CCSS: F-IF.9)
[8] For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
[9] both positive and negative. (CCSS: F-BF.3)
[10] Include recognizing even and odd functions from their graphs and algebraic expressions for them. (CCSS: F-BF.3)
[11] Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
For example, f(x) =2 x^3 or f(x) = (x+1)/(x–1) for x ≠ 1. (CCSS: F-BF.4a)
[12] include reading these from a table. (CCSS: F-LE.2)
[13] For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. (CCSS: A-SSE.1b)
[14] For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (CCSS: A-SSE.2)
[15] For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. (A-SSE.3c)
[16] For example, calculate mortgage payments. (CCSS: A-SSE.4)
[17] For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). (CCSS: A-APR.2)
[18] For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. (CCSS: A-APR.4)
[19] write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. (CCSS: A-APR.6)
[20] Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (CCSS: A-CED.1)
[21] For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (CCSS: A-CED.3)
[22] For example, rearrange Ohm's law V = IR to highlight resistance R. (CCSS: A-CED.4)
[23] e.g., for $x^2 = 49$. (CCSS: A-REI.4b)
[24] e.g., with graphs. (CCSS: A-REI.6)
[25] For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. (CCSS: A-REI.7)
[26] which could be a line. (CCSS: A-REI.10)
[27] Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (CCSS: A-REI.11)
[28] e.g., using technology to graph the functions, make tables of values, or find successive approximations. (CCSS: A-REI.11)

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Describe the connections between proportional relationships, lines, and linear equations. (CCSS: 8.EE) b. Graph proportional relationships, interpreting the unit rate as the slope of the graph. (CCSS: 8.EE.5) c. Compare two different proportional relationships represented in different ways.1 (CCSS: 8.EE.5) d. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (CCSS: 8.EE.6) e. Derive the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. (CCSS: 8.EE.6)	**Inquiry Questions:** 1. How can different representations of linear patterns present different perspectives of situations? 2. How can a relationship be analyzed with tables, graphs, and equations? 3. Why is one variable dependent upon the other in relationships? **Relevance and Application:** 1. Fluency with different representations of linear patterns allows comparison and contrast of linear situations such as service billing rates from competing companies or simple interest on savings or credit. 2. Understanding slope as rate of change allows individuals to develop and use a line of best fit for data that appears to be linearly related. 3. The ability to recognize slope and y-intercept of a linear function facilitates graphing the function or writing an equation that describes the function. **Nature of Mathematics:** 1. Mathematicians represent functions in multiple ways to gain insights into the relationships they model. 2. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Match a graphical representation in the first quadrant with a table of values.	**Content based access skills:** 1. Attaching meaning to mathematical functions symbols 2. Applying technology to solve mathematical equations 3. Working cooperatively with others during mathematical activities 4. Manipulating mathematical materials and equipment

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

2. Properties of algebra and equality are used to solve linear equations and systems of equations

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Solve linear equations in one variable. (CCSS: 8.EE.7) i. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.2 (CCSS: 8.EE.7a) ii. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (CCSS: 8.EE.7b) b. Analyze and solve pairs of simultaneous linear equations. (CCSS: 8.EE.8) i. Explain that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (CCSS: 8.EE.8a) ii. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.3 (CCSS: 8.EE.8b) iii. Solve real-world and mathematical problems leading to two linear equations in two variables.4 (CCSS: 8.EE.8c)	**Inquiry Questions:** 2. What makes a solution strategy both efficient and effective? 3. How is it determined if multiple solutions to an equation are valid? 4. How does the context of the problem affect the reasonableness of a solution? 5. Why can two equations be added together to get another true equation? **Relevance and Application:** 1. The understanding and use of equations, inequalities, and systems of equations allows for situational analysis and decision-making. For example, it helps people choose cell phone plans, calculate credit card interest and payments, and determine health insurance costs. 2. Recognition of the significance of the point of intersection for two linear equations helps to solve problems involving two linear rates such as determining when two vehicles traveling at constant speeds will be in the same place, when two calling plans cost the same, or the point when profits begin to exceed costs. **Nature of Mathematics:** 1. Mathematics involves visualization. 2. Mathematicians use tools to create visual representations of problems and ideas that reveal relationships and meaning. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians use appropriate tools strategically. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Find the solution of a simple linear equation involving addition (one step) (e.g. 3 + ? = 5).	**Content based access skills:** 1. Selecting appropriate technology to solve mathematical equations 2. Sequencing mathematical terms

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Define, evaluate, and compare functions. (CCSS: 8.F) i. Define a function as a rule that assigns to each input exactly one output.5 (CCSS: 8.F.1) ii. Show that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (CCSS: 8.F.1) iii. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).6 (CCSS: 8.F.2) iv. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line. (CCSS: 8.F.3) v. Give examples of functions that are not linear.7 b. Use functions to model relationships between quantities. (CCSS: 8.F) i. Construct a function to model a linear relationship between two quantities. (CCSS: 8.F.4) ii. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. (CCSS: 8.F.4) iii. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (CCSS: 8.F.4) iv. Describe qualitatively the functional relationship between two quantities by analyzing a graph.8 (CCSS: 8.F.5) v. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (CCSS: 8.F.5) vi. Analyze how credit and debt impact personal financial goals (PFL)	**Inquiry Questions:** 1. How can change best be represented mathematically? 2. Why are patterns and relationships represented in multiple ways? 3. What properties of a function make it a linear function? **Relevance and Application:** 1. Recognition that non-linear situations is a clue to non-constant growth over time helps to understand such concepts as compound interest rates, population growth, appreciations, and depreciation. 2. Linear situations allow for describing and analyzing the situation mathematically such as using a line graph to represent the relationships of the circumference of circles based on diameters. **Nature of Mathematics:** 1. Mathematics involves multiple points of view. 2. Mathematicians look at mathematical ideas arithmetically, geometrically, analytically, or through a combination of these approaches. 3. Mathematicians look for and make use of structure. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Extend a linear pattern by supplying the next element. II. Identify the rule within a simple linear pattern (addition or subtraction). III. Find the ordered pair that identifies a point in the first quadrant where the axes extend up to 10.	**Content based access skills:** 1. Attaching meaning to mathematical functions symbols 2. Working cooperatively with others during mathematical activities 3. Manipulating mathematical materials and equipment 4. Accessing and using communication system to respond to mathematical problems

Standard: 2. Patterns, Functions, and Algebraic Structures
Eighth Grade

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[1] For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. (CCSS: 8.EE.5)
[2] Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (CCSS: 8.EE.6a)
[3] For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6. (CCSS: 8.EE.8b)
[4] For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (CCSS: 8.EE.8c)
[5] Function notation is not required in 8th grade. (CCSS: 8.F.1[1])
[6] For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (CCSS: 8.F.2)
[7] For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line. (CCSS: 8.F.3)
[8] e.g., where the function is increasing or decreasing, linear or nonlinear. (CCSS: 8.F.5)

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Properties of arithmetic can be used to generate equivalent expressions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use properties of operations to generate equivalent expressions. (CCSS: 7.EE) i. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (CCSS: 7.EE.1) ii. Demonstrate that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.1 (CCSS: 7.EE.2)	**Inquiry Questions:** 1. How do symbolic transformations affect an equation or expression? 2. How is it determined that two algebraic expressions are equivalent? **Relevance and Application:** 1. The ability to recognize and find equivalent forms of an equation allows the transformation of equations into the most useful form such as adjusting the density formula to calculate for volume or mass. **Nature of Mathematics:** 1. Mathematicians abstract a problem by representing it as an equation. They travel between the concrete problem and the abstraction to gain insights and find solutions. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Generate multiplication fact families when given whole number single digit components.	**Content based access skills:** 1. Sequencing mathematical terms 2. Accessing and using communication system to respond to mathematical problems

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

2. Equations and expressions model quantitative relationships and phenomena

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form,2 using tools strategically. (CCSS: 7.EE.3) b. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies.3 (CCSS: 7.EE.3) c. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (CCSS: 7.EE.4) i. Fluently solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. (CCSS: 7.EE.4a) ii. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.4 (CCSS: 7.EE.4a) iii. Solve word problems5 leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. (CCSS: 7.EE.4b) iv. Graph the solution set of the inequality and interpret it in the context of the problem. (CCSS: 7.EE.4b)	**Inquiry Questions:** 1. Do algebraic properties work with numbers or just symbols? Why? 2. Why are there different ways to solve equations? 3. How are properties applied in other fields of study? 4. Why might estimation be better than an exact answer? 5. When might an estimate be the only possible answer? **Relevance and Application:** 1. Procedural fluency with algebraic methods allows use of linear equations and inequalities to solve problems in fields such as banking, engineering, and insurance. For example, it helps to calculate the total value of assets or find the acceleration of an object moving at a linearly increasing speed. 2. Comprehension of the structure of equations allows one to use spreadsheets effectively to solve problems that matter such as showing how long it takes to pay off debt, or representing data collected from science experiments. 3. Estimation with rational numbers enables quick and flexible decision-making in daily life. For example, determining how many batches of a recipe can be made with given ingredients, how many floor tiles to buy with given dimensions, the amount of carpeting needed for a room, or fencing required for a backyard. **Nature of Mathematics:** 1. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Solve a real world problem using manipulatives/tools and a completed table of a simple linear function (y = mx).	**Content based access skills:** 1. Attaching meaning to mathematical functions symbols 2. Manipulating mathematical materials and equipment

Standard: 2. Patterns, Functions, and Algebraic Structures
Seventh Grade

[1] For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05." (CCSS: 7.EE.2)
[2] whole numbers, fractions, and decimals. (CCSS: 7.EE.3)
[3] For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. (CCSS: 7.EE.3)
[4] For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? (CCSS: 7.EE.4a)
[5] For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions. (CCSS: 7.EE.4b)

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Algebraic expressions can be used to generalize properties of arithmetic

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Write and evaluate numerical expressions involving whole-number exponents. (CCSS: 6.EE.1) b. Write, read, and evaluate expressions in which letters stand for numbers. (CCSS: 6.EE.2) i. Write expressions that record operations with numbers and with letters standing for numbers.1 (CCSS: 6.EE.2a) ii. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient) and describe one or more parts of an expression as a single entity.2 (CCSS: 6.EE.2b) iii. Evaluate expressions at specific values of their variables including expressions that arise from formulas used in real-world problems.3 (CCSS: 6.EE.2c) iv. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). (CCSS: 6.EE.2c) c. Apply the properties of operations to generate equivalent expressions.4 (CCSS: 6.EE.3) d. Identify when two expressions are equivalent.5 (CCSS: 6.EE.4)	**Inquiry Questions:** 1. If we didn't have variables, what would we use? 2. What purposes do variable expressions serve? 3. What are some advantages to being able to describe a pattern using variables? 4. Why does the order of operations exist? 5. What other tasks/processes require the use of a strict order of steps? **Relevance and Application:** 1. The simplification of algebraic expressions allows one to communicate mathematics efficiently for use in a variety of contexts. 2. Using algebraic expressions we can efficiently expand and describe patterns in spreadsheets or other technologies. **Nature of Mathematics:** 1. Mathematics can be used to show that things that seem complex can be broken into simple patterns and relationships. 2. Mathematics can be expressed in a variety of formats. 3. Mathematicians reason abstractly and quantitatively. (MP) 4. Mathematicians look for and make use of structure. (MP) 5. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify the rule for an arithmetic pattern (n + c) with a sum up to 10, using manipulatives/tools. II. Find x for a simple addition equation: (x + a constant up to 5 = sum up to 10) using manipulatives.	**Content based access skills:** 1. Recognizing numerical patterns 2. Understanding mathematical operation symbols 3. Sequencing mathematical terms

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

2. Variables are used to represent unknown quantities within equations and inequalities

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Describe solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? (CCSS: 6.EE.5) b. Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (CCSS: 6.EE.5) c. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. (CCSS: 6.EE.6) i. Recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (CCSS: 6.EE.6) d. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (CCSS: 6.EE.7) e. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. (CCSS: 6.EE.8) f. Show that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (CCSS: 6.EE.8) g. Represent and analyze quantitative relationships between dependent and independent variables. (CCSS: 6.EE) i. Use variables to represent two quantities in a real-world problem that change in relationship to one another. (CCSS: 6.EE.9) ii. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. (CCSS: 6.EE.9) iii. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.6 (CCSS: 6.EE.9)	**Inquiry Questions:** 1. Do all equations have exactly one unique solution? Why? 2. How can you determine if a variable is independent or dependent? **Relevance and Application:** 1. Variables allow communication of big ideas with very few symbols. For example, d = r * t is a simple way of showing the relationship between the distance one travels and the rate of speed and time traveled, and $C = \pi d$ expresses the relationship between circumference and diameter of a circle. 2. Variables show what parts of an expression may change compared to those parts that are fixed or constant. For example, the price of an item may be fixed in an expression, but the number of items purchased may change. **Nature of Mathematics:** 1. Mathematicians use graphs and equations to represent relationships among variables. They use multiple representations to gain insights into the relationships between variables. 2. Mathematicians can think both forward and backward through a problem. An equation is like the end of a story about what happened to a variable. By reading the story backward, and undoing each step, mathematicians can find the value of the variable. 3. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Complete output values in a table were the rule is 2n, 5n, or 10n.	**Content based access skills:** 1. Recognizing numerical patterns 2. Understanding mathematical operation symbols 3. Accessing and using communication system to respond to mathematical problems

[1] For example, express the calculation "Subtract y from 5" as 5 – y. (CCSS: 6.EE.2a)
[2] For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. (CCSS: 6.EE.2b)
[3] For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2. (CCSS: 6.EE.2c)
[4] For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. (CCSS: 6.EE.3)
[5] i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. Reason about and solve one-variable equations and inequalities. (CCSS: 6.EE.4)
[6] For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time. (CCSS: 6.EE.9)

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. Number patterns are based on operations and relationships

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Generate two numerical patterns using given rules. (CCSS: 5.OA.3) b. Identify apparent relationships between corresponding terms. (CCSS: 5.OA.3) c. Form ordered pairs consisting of corresponding terms from the two patterns, and graphs the ordered pairs on a coordinate plane.1 (CCSS: 5.OA.3) d. Explain informally relationships between corresponding terms in the patterns. (CCSS: 5.OA.3) e. Use patterns to solve problems including those involving saving and checking accounts2 (PFL) f. Explain, extend, and use patterns and relationships in solving problems, including those involving saving and checking accounts such as understanding that spending more means saving less (PFL)	**Inquiry Questions:** 1. How do you know when there is a pattern? 2. How are patterns useful? **Relevance and Application:** 1. The use of a pattern of elapsed time helps to set up a schedule. For example, classes are each 50 minutes with 5 minutes between each class. 2. The ability to use patterns allows problem-solving. For example, a rancher needs to know how many shoes to buy for his horses, or a grocer needs to know how many cans will fit on a set of shelves. **Nature of Mathematics:** 1. Mathematicians use creativity, invention, and ingenuity to understand and create patterns. 2. The search for patterns can produce rewarding shortcuts and mathematical insights. 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians model with mathematics. (MP) 5. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Given a rule, generate a numerical or representative pattern (i.e. add two, subtract one).	**Content based access skills:** 1. Recognizing and reproducing a mathematical pattern 2. Accessing and using communication system to respond to mathematical problems 3. Selecting appropriate technology to solve mathematical equations

[1] For example, given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms and the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. (CCSS: 5.OA.3)
[2] such as the pattern created when saving $10 a month

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. Number patterns and relationships can be represented by symbols

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Generate and analyze patterns and identify apparent features of the pattern that were not explicit in the rule itself.1 (CCSS: 4.OA.5) i. Use number relationships to find the missing number in a sequence ii. Use a symbol to represent and find an unknown quantity in a problem situation iii. Complete input/output tables iv. Find the unknown in simple equations b. Apply concepts of squares, primes, composites, factors, and multiples to solve problems i. Find all factor pairs for a whole number in the range 1–100. (CCSS: 4.OA.4) ii. Recognize that a whole number is a multiple of each of its factors. (CCSS: 4.OA.4) iii. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. (CCSS: 4.OA.4) iv. Determine whether a given whole number in the range 1–100 is prime or composite. (CCSS: 4.OA.4)	**Inquiry Questions:** 1. What characteristics can be used to classify numbers into different groups? 2. How can we predict the next element in a pattern? 3. Why do we use symbols to represent missing numbers? 4. Why is finding an unknown quantity important? **Relevance and Application:** 1. Use of an input/output table helps to make predictions in everyday contexts such as the number of beads needed to make multiple bracelets or number of inches of expected growth. 2. Symbols help to represent situations from everyday life with simple equations such as finding how much additional money is needed to buy a skateboard, determining the number of players missing from a soccer team, or calculating the number of students absent from school. 3. Comprehension of the relationships between primes, composites, multiples, and factors develop number sense. The relationships are used to simplify computations with large numbers, algebraic expressions, and division problems, and to find common denominators. **Nature of Mathematics:** 1. Mathematics involves pattern seeking. 2. Mathematicians use patterns to simplify calculations. 3. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Count by 2s to 20 using sets of manipulatives. II. Apply a simple rule (___+ a constant less than 3) to fill in a missing element in a table using manipulatives. III. Identify one missing element in an ABCABC pattern. IV. Extend a repeating ABCABC pattern by two elements.	**Content based access skills:** 1. Responding to others in reproducing and modeling mathematical tasks 2. Expressing personal preferences and choices related to patterns

Standard: 2. Patterns, Functions, and Algebraic Structures
Fourth Grade

[1] For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. (CCSS: 4.OA.5)

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:	
Grade Level Expectation: PRESCHOOL THROUGH THIRD GRADE	
Concepts and skills students master:	
Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: **Expectations for this standard are integrated into the other standards at preschool through third grade.**	**Inquiry Questions:** **Relevance and Application:** **Nature of Physical Education:**

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: High School

Concepts and skills students master:

1. Visual displays and summary statistics condense the information in data sets into usable knowledge

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Summarize, represent, and interpret data on a single count or measurement variable. (CCSS: S-ID) i. Represent data with plots on the real number line (dot plots, histograms, and box plots). (CCSS: S-ID.1) ii. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (CCSS: S-ID.2) iii. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (CCSS: S-ID.3) iv. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and identify data sets for which such a procedure is not appropriate. (CCSS: S-ID.4) v. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. (CCSS: S-ID.4) b. Summarize, represent, and interpret data on two categorical and quantitative variables. (CCSS: S-ID) i. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data1 (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (CCSS: S-ID.5) ii. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (CCSS: S-ID.6) 1. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (CCSS: S-ID.6a) 2. Informally assess the fit of a function by plotting and analyzing residuals. (CCSS: S-ID.6b) 3. Fit a linear function for a scatter plot that suggests a linear association. (CCSS: S-ID.6c) c. Interpret linear models. (CCSS: S-ID) i. Interpret the slope2 and the intercept3 of a linear model in the context of the data. (CCSS: S-ID.7) ii. Using technology, compute and interpret the correlation coefficient of a linear fit. (CCSS: S-ID.8) iii. Distinguish between correlation and causation. (CCSS: S-ID.9)	**Inquiry Questions:** 1. What makes data meaningful or actionable? 2. Why should attention be paid to an unexpected outcome? 3. How can summary statistics or data displays be accurate but misleading? **Relevance and Application:** 1. Facility with data organization, summary, and display allows the sharing of data efficiently and collaboratively to answer important questions such as is the climate changing, how do people think about ballot initiatives in the next election, or is there a connection between cancers in a community? **Nature of Mathematics:** 1. Mathematicians create visual and numerical representations of data to reveal relationships and meaning hidden in the raw data. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians model with mathematics. (MP) 4. Mathematicians use appropriate tools strategically. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Determine the whole number mean (average) using manipulatives and a frequency chart (data set up to 10). II. Describe the range of a data set (beginning and end points). III. Explore the correlation between two variables given a scatter plot (e.g. student weight increases with height). IV. Determine the best value by using a display of quantity and price with manipulative/tools.	**Content based access skills:** 1. Attaching meaning to mathematical symbols 2. Applying technology to solve mathematical equations 3. Manipulating mathematical materials and equipment

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking

Grade Level Expectation: High School

Concepts and skills students master:

2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Understand and evaluate random processes underlying statistical experiments. (CCSS: S-IC) i. Describe statistics as a process for making inferences about population parameters based on a random sample from that population. (CCSS: S-IC.1) ii. Decide if a specified model is consistent with results from a given data-generating process.4 (CCSS: S-IC.2) b. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. (CCSS: S-IC) i. Identify the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (CCSS: S-IC.3) ii. Use data from a sample survey to estimate a population mean or proportion. (CCSS: S-IC.4) iii. Develop a margin of error through the use of simulation models for random sampling. (CCSS: S-IC.4) iv. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (CCSS: S-IC.5) v. Define and explain the meaning of significance, both statistical (using *p*-values) and practical (using effect size). vi. Evaluate reports based on data. (CCSS: S-IC.6)	**Inquiry Questions:** 1. How can the results of a statistical investigation be used to support an argument? 2. What happens to sample-to-sample variability when you increase the sample size? 3. When should sampling be used? When is sampling better than using a census? 4. Can the practical significance of a given study matter more than statistical significance? Why is it important to know the difference? 5. Why is the margin of error in a study important? 6. How is it known that the results of a study are not simply due to chance? **Relevance and Application:** 1. Inference and prediction skills enable informed decision-making based on data such as whether to stop using a product based on safety concerns, or whether a political poll is pointing to a trend. **Nature of Mathematics:** 1. Mathematics involves making conjectures, gathering data, recording results, and making multiple tests. 2. Mathematicians are skeptical of apparent trends. They use their understanding of randomness to distinguish meaningful trends from random occurrences. 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians model with mathematics. (MP) 5. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Differentiate between fact and opinion based on research data (e.g. "9 out of 10 dentists recommend Brand A toothpaste" vs. "Brand A is the best toothpaste"). II. Generate an appropriate survey question for a given research question.	**Content based access skills:** 1. Expressing and understanding that information can be fact or opinion in relation to math 2. Working cooperatively with others during mathematical activities

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: High School

Concepts and skills students master:

3. Probability models outcomes for situations in which there is inherent randomness

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Understand independence and conditional probability and use them to interpret data. (CCSS: S-CP) i. Describe events as subsets of a sample space5 using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events.6 (CCSS: S-CP.1) ii. Explain that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (CCSS: S-CP.2) iii. Using the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), interpret the independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. (CCSS: S-CP.3) iv. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.*7* (CCSS: S-CP.4) v. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.8 (CCSS: S-CP.5) b. Use the rules of probability to compute probabilities of compound events in a uniform probability model. (CCSS: S-CP) i. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. (CCSS: S-CP.6) ii. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. (CCSS: S-CP.7) c. Analyze* the cost of insurance as a method to offset the risk of a situation (PFL) **Indicates a part of the standard connected to the mathematical practice of Modeling.*	**Inquiry Questions:** 1. Can probability be used to model all types of uncertain situations? For example, can the probability that the 50th president of the United States will be female be determined? 2. How and why are simulations used to determine probability when the theoretical probability is unknown? 3. How does probability relate to obtaining insurance? (PFL) **Relevance and Application:** 1. Comprehension of probability allows informed decision-making, such as whether the cost of insurance is less than the expected cost of illness, when the deductible on car insurance is optimal, whether gambling pays in the long run, or whether an extended warranty justifies the cost. (PFL) 2. Probability is used in a wide variety of disciplines including physics, biology, engineering, finance, and law. For example, employment discrimination cases often present probability calculations to support a claim. **Nature of Mathematics:** 1. Some work in mathematics is much like a game. Mathematicians choose an interesting set of rules and then play according to those rules to see what can happen. 2. Mathematicians explore randomness and chance through probability. 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Interpret independent and dependent events. II. Use the terms "highly likely", "likely", and "not likely" to evaluate statements of cause and effect. III. Determine the possible combinations when given two variables (up to three options in each variable) (e.g. Outfits given shirts and pants, kinds of sandwiches given bread and meat choices, kinds of pizza given toppings).	**Content based access skills:** 1. Expressing personal preferences and choices related to cause and effect 2. Attaching meaning to mathematical functions symbols 3. Manipulating mathematical materials and equipment

[1] including joint, marginal, and conditional relative frequencies.
[2] rate of change. (CCSS: S-ID.7)
[3] constant term. (CCSS: S-ID.7)
[4] e.g., using simulation. (CCSS: S-IC.2)
For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (CCSS: S-IC.2)
[5] the set of outcomes. (CCSS: S-CP.1)
[6] "or," "and," "not". (CCSS: S-CP.1)
[7] For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (CCSS: S-CP.4)
[8] For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (CCSS: S-CP.5)

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. (CCSS: 8.SP.1) b. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (CCSS: 8.SP.1) c. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.1 (CCSS: 8.SP.2) d. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.2 (CCSS: 8.SP.3) e. Explain patterns of association seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. (CCSS: 8.SP.4) i. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. (CCSS: 8.SP.4) ii. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.3 (CCSS: 8.SP.4)	**Inquiry Questions:** 1. How is it known that two variables are related to each other? 2. How is it known that an apparent trend is just a coincidence? 3. How can correct data lead to incorrect conclusions? 4. How do you know when a credible prediction can be made? **Relevance and Application:** 1. The ability to analyze and interpret data helps to distinguish between false relationships such as developing superstitions from seeing two events happen in close succession versus identifying a credible correlation. 2. Data analysis provides the tools to use data to model relationships, make predictions, and determine the reasonableness and limitations of those predictions. For example, predicting whether staying up late affects grades, or the relationships between education and income, between income and energy consumption, or between the unemployment rate and GDP. **Nature of Mathematics:** 1. Mathematicians discover new relationship embedded in information. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Indicate a general trend on a line graph (increasing, decreasing or staying the same).	**Content based access skills:** 1. Expressing an understanding of increasing and decreasing in relation to numbers 2. Accessing and using a communication system to respond to mathematical problems

[1] Know that straight lines are widely used to model relationships between two quantitative variables. (CCSS: 8.SP.2)
[2] For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. (CCSS: 8.SP.3)
[3] For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores? (CCSS: 8.SP.4)

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Statistics can be used to gain information about populations by examining samples

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use random sampling to draw inferences about a population. (CCSS: 7.SP) i. Explain that generalizations about a population from a sample are valid only if the sample is representative of that population. (CCSS: 7.SP.1) ii. Explain that random sampling tends to produce representative samples and support valid inferences. (CCSS: 7.SP.1) iii. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. (CCSS: 7.SP.2) iv. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.1 (CCSS: 7.SP.2) b. Draw informal comparative inferences about two populations. (CCSS: 7.SP) i. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.2 (CCSS: 7.SP.3) ii. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.3 (CCSS: 7.SP.4)	**Inquiry Questions:** 1. How might the sample for a survey affect the results of the survey? 2. How do you distinguish between random and bias samples? 3. How can you declare a winner in an election before counting all the ballots? **Relevance and Application:** 1. The ability to recognize how data can be biased or misrepresented allows critical evaluation of claims and avoids being misled. For example, data can be used to evaluate products that promise effectiveness or show strong opinions. 2. Mathematical inferences allow us to make reliable predictions without accounting for every piece of data. **Nature of Mathematics:** 1. Mathematicians are informed consumers of information. They evaluate the quality of data before using it to make decisions. 2. Mathematicians use appropriate tools strategically. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify whether the information from a small, obviously biased sample can be generalized to the entire population. II. Draw a conclusion from a graphical representation of survey results (e.g. most students prefer chocolate ice cream, the cafeteria buys more chocolate ice cream than other flavors). III. Identify the whole number median of a set of single digit numbers using tools and manipulatives.	**Content based access skills:** 1. Manipulating mathematical materials and equipment 2. Attaching meaning to mathematical graphs 3. Accessing and using communication system to respond to mathematical problems

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

2. Mathematical models are used to determine probability

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Explain that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.4 (CCSS: 7.SP.5) b. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.5 (CCSS: 7.SP.6) c. Develop a probability model and use it to find probabilities of events. (CCSS: 7.SP.7) i. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (CCSS: 7.SP.7) ii. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.6 (CCSS: 7.SP.7a) iii. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.7 (CCSS: 7.SP.7b) d. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (CCSS: 7.SP.8) i. Explain that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (CCSS: 7.SP.8a) ii. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. (CCSS: 7.SP.8b) iii. For an event8 described in everyday language identify the outcomes in the sample space which compose the event. (CCSS: 7.SP.8b) iv. Design and use a simulation to generate frequencies for compound events.9 (CCSS: 7.SP.8c)	**Inquiry Questions:** 1. Why is it important to consider all of the possible outcomes of an event? 2. Is it possible to predict the future? How? 3. What are situations in which probability cannot be used? **Relevance and Application:** 1. The ability to efficiently and accurately count outcomes allows systemic analysis of such situations as trying all possible combinations when you forgot the combination to your lock or deciding to find a different approach when there are too many combinations to try; or counting how many lottery tickets you would have to buy to play every possible combination of numbers. 2. The knowledge of theoretical probability allows the development of winning strategies in games involving chance such as knowing if your hand is likely to be the best hand or is likely to improve in a game of cards. **Nature of Mathematics:** 1. Mathematicians approach problems systematically. When the number of possible outcomes is small, each outcome can be considered individually. When the number of outcomes is large, a mathematician will develop a strategy to consider the most important outcomes such as the most likely outcomes, or the most dangerous outcomes. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify the most likely event given a circle or bar graph.	**Content based access skills:** 1. Attaching meaning to mathematical graphs 2. Manipulating mathematical materials and equipment

Standard: 3. Data Analysis, Statistics, and Probability
Seventh Grade

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[1] For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. (CCSS: 7.SP.2)
[2] For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. (CCSS: 7.SP.3)
[3] For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. (CCSS: 7.SP.4)
[4] Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (CCSS: 7.SP.5)
[5] For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. (CCSS: 7.SP.6)
[6] For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. (CCSS: 7.SP.7a)
[7] For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? (CCSS: 7.SP.7b)
[8] e.g., "rolling double sixes" (CCSS: 7.SP.8b)
[9] For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood? (CCSS: 7.SP.8c)

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.1 (CCSS: 6.SP.1) b. Demonstrate that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (CCSS: 6.SP.2) c. Explain that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (CCSS: 6.SP.3) d. Summarize and describe distributions. (CCSS: 6.SP) i. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (CCSS: 6.SP.4) ii. Summarize numerical data sets in relation to their context. (CCSS: 6.SP.5) 1. Report the number of observations. (CCSS: 6.SP.5a) 2. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. (CCSS: 6.SP.5b) 3. Give quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (CCSS: 6.SP.5c) 4. Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (CCSS: 6.SP.5d)	**Inquiry Questions:** 1. Why are there so many ways to describe data? 2. When is one data display better than another? 3. When is one statistical measure better than another? 4. What makes a good statistical question? **Relevance and Application:** 1. Comprehension of how to analyze and interpret data allows better understanding of large and complex systems such as analyzing employment data to better understand our economy, or analyzing achievement data to better understand our education system. 2. Different data analysis tools enable the efficient communication of large amounts of information such as listing all the student scores on a state test versus using a box plot to show the distribution of the scores. **Nature of Mathematics:** 1. Mathematicians leverage strategic displays to reveal data. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Select an appropriate population for a given research question (e.g. children favorite type of cartoon, adults type of car they drive). II. Display numerical data in plots on a number-line (0 - 20) from a given set of data. III. Determine the mode of a set of data using a pre-populated frequency chart (where only one mode exists).	**Content based access skills:** 1. Expressing an understanding that information gathered can be connected to a number 2. Applying technology to solve mathematical equations 3. Working cooperatively with others during mathematical activities

[1] For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages. (CCSS: 6.SP.1)

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. Visual displays are used to interpret data

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Represent and interpret data. (CCSS: 5.MD) i. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). (CCSS: 5.MD.2) ii. Use operations on fractions for this grade to solve problems involving information presented in line plots.1 (CCSS: 5.MD.2)	**Inquiry Questions:** 1. How can you make sense of the data you collect? **Relevance and Application:** 1. The collection and analysis of data provides understanding of how things work. For example, measuring the temperature every day for a year helps to better understand weather. **Nature of Mathematics:** 1. Mathematics helps people collect and use information to make good decisions. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Display previously gathered data into provided labeled graph or table. II. Generate and record a data set using a chance device (die, coin, and spinner).	**Content based access skills:** 1. Working collaboratively with a group around mathematical concepts 2. Expressing an understanding that money has a value and can be exchanged for goods and services 3. Attaching meaning to mathematical graphs 4. Manipulating mathematical materials and equipment

[1] For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. (CCSS: 5.MD.2)

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. Visual displays are used to represent data

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). (CCSS: 4.MD.4) b. Solve problems involving addition and subtraction of fractions by using information presented in line plots.1 (CCSS: 4.MD.4)	**Inquiry Questions:** 1. What can you learn by collecting data? 2. What can the shape of data in a display tell you? **Relevance and Application:** 1. The collection and analysis of data provides understanding of how things work. For example, measuring the weather every day for a year helps to better understand weather. **Nature of Mathematics:** 1. Mathematics helps people use data to learn about the world. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Interpret data from a pictograph to answer "how many" (answer derived from one column or data set)	**Content based access skills:** 1. Attaching meaning to mathematical symbols on a graph 2. Accessing and using communication system to respond to mathematical problems

Standard: 3. Data Analysis, Statistics, and Probability
Fourth Grade

[1] For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. (CCSS: 4.MD.4)

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. Visual displays are used to describe data

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Represent and interpret data. (CCSS: 3.MD) i. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. (CCSS: 3.MD.3) ii. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.1 (CCSS: 3.MD.3) iii. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. (CCSS: 3.MD.4)	**Inquiry Questions:** 1. What can data tell you about your class or school? 2. How do data displays help us understand information? **Relevance and Application:** 1. The collection and use of data provides better understanding of people and the world such as knowing what games classmates like to play, how many siblings friends have, or personal progress made in sports. **Nature of Mathematics:** 1. Mathematical data can be represented in both static and animated displays. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Analyze data to identify "most/least" and "more/less" based on a data set. II. Investigate the results of using a chance device (spinner, die, coin etc).	**Content based access skills:** 1. Attaching meaning to mathematical terms of most/least and more/less 2. Manipulating mathematical materials and equipment

Standard: 3. Data Analysis, Statistics, and Probability
Third Grade

[1] For example, draw a bar graph in which each square in the bar graph might represent 5 pets. (CCSS: 3.MD.3)

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Second Grade

Concepts and skills students master:

1. Visual displays of data can be constructed in a variety of formats to solve problems

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Represent and interpret data. (CCSS: 2.MD) i. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (CCSS: 2.MD.9) ii. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. (CCSS: 2.MD.10) iii. Solve simple put together, take-apart, and compare problems using information presented in picture and bar graphs. (CCSS: 2.MD.10)	**Inquiry Questions:** 1. What are the ways data can be displayed? 2. What can data tell you about the people you survey? 3. What makes a good survey question? **Relevance and Application:** 1. People use data to describe the world and answer questions such as how many classmates are buying lunch today, how much it rained yesterday, or in which month are the most birthdays. **Nature of Mathematics:** 1. Mathematics can be displayed as symbols. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians model with mathematics. (MP) 4. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Use a pictograph to answer a question (most/least, more/less).	**Content based access skills:** 1. Attaching meaning to mathematical pictographs 2. Expressing personal preferences and choices related to mathematical terms of more/less, most/least

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: First Grade

Concepts and skills students master:

1. Visual displays of information can used to answer questions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Represent and interpret data. (CCSS: 1.MD) i. Organize, represent, and interpret data with up to three categories. (CCSS: 1.MD.4) ii. Ask and answer questions about the total number of data points how many in each category, and how many more or less are in one category than in another. (CCSS: 1.MD.4)	**Inquiry Questions:** 1. What kinds of questions generate data? 2. What questions can be answered by a data representation? **Relevance and Application:** 1. People use graphs and charts to communicate information and learn about a class or community such as the kinds of cars people drive, or favorite ice cream flavors of a class. **Nature of Mathematics:** 1. Mathematicians organize and explain random information 2. Mathematicians model with mathematics. (MP)

Extended Evidence Outcomes	Extended Readiness Competencies
With appropriate supports, students can: I. Organize data with up to 6 data points into two categories. (e.g. dogs/cats, yes/no)	**Content based access skills:** 1. Working cooperatively with others during mathematical activities 2. Engaging in sustained participation in mathematical activities 3. Manipulating mathematical materials and equipment

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

Grade Level Expectation: PRESCHOOL AND KINDERGARTEN

Concepts and skills students master:

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: **Expectations for this standard are integrated into the other standards at preschool through kindergarten.**	**Inquiry Questions:** **Relevance and Application:** **Nature of Physical Education:**

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: High School

Concepts and skills students master:

1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Experiment with transformations in the plane. (CCSS: G-CO) i. State precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (CCSS: G-CO.1) ii. Represent transformations in the plane using1 appropriate tools. (CCSS: G-CO.2) iii. Describe transformations as functions that take points in the plane as inputs and give other points as outputs. (CCSS: G-CO.2) iv. Compare transformations that preserve distance and angle to those that do not.2 (CCSS: G-CO.2) v. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (CCSS: G-CO.3) vi. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (CCSS: G-CO.4) vii. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using appropriate tools.3 (CCSS: G-CO.5) viii. Specify a sequence of transformations that will carry a given figure onto another. (CCSS: G-CO.5) b. Understand congruence in terms of rigid motions. (CCSS: G-CO) i. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure. (CCSS: G-CO.6) ii. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (CCSS: G-CO.6) iii. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (CCSS: G-CO.7) iv. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (CCSS: G-CO.8) c. Prove geometric theorems. (CCSS: G-CO) i. Prove theorems about lines and angles.4 (CCSS: G-CO.9) ii. Prove theorems about triangles.5 (CCSS: G-CO.10) iii. Prove theorems about parallelograms.6 (CCSS: G-CO.11) d. Make geometric constructions. (CCSS: G-CO) i. Make formal geometric constructions7 with a variety of tools and methods.8 (CCSS: G-CO.12) ii. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (CCSS: G-CO.13)	**Inquiry Questions:** 1. What happens to the coordinates of the vertices of shapes when different transformations are applied in the plane? 2. How would the idea of congruency be used outside of mathematics? 3. What does it mean for two things to be the same? Are there different degrees of "sameness?" 4. What makes a good definition of a shape? **Relevance and Application:** 1. Comprehension of transformations aids with innovation and creation in the areas of computer graphics and animation. **Nature of Mathematics:** 1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians attend to precision. (MP) 4. Mathematicians look for and make use of structure. (MP)

Extended Evidence Outcomes	Extended Readiness Competencies
With appropriate supports, students can: I. Identify parallel, intersecting and perpendicular lines. II. Demonstrate congruence of two geometric shapes using translation. III. Complete an if/then statement related to real life situations. IV. Create geometric shapes using construction tools including technology. V. Identify the midpoint on a line segment.	**Content based access skills:** 1. Attaching meaning to mathematical symbols for parallel, intersecting and perpendicular 2. Working cooperatively with others during mathematical activities 3. Manipulating mathematical materials and equipment including geometric shapes 4. Expressing an understanding of cause and effect

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: High School

Concepts and skills students master:

2. Concepts of similarity are foundational to geometry and its applications

Evidence Outcomes

Students can:

a. Understand similarity in terms of similarity transformations. (CCSS: G-SRT)
   i. Verify experimentally the properties of dilations given by a center and a scale factor. (CCSS: G-SRT.1)
      1. Show that a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. (CCSS: G-SRT.1a)
      2. Show that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. (CCSS: G-SRT.1b)
   ii. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar. (CCSS: G-SRT.2)
   iii. Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (CCSS: G-SRT.2)
   iv. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (CCSS: G-SRT.3)

b. Prove theorems involving similarity. (CCSS: G-SRT)
   i. Prove theorems about triangles.9 (CCSS: G-SRT.4)
   ii. Prove that all circles are similar. (CCSS: G-C.1)
   iii. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (CCSS: G-SRT.5)

c. Define trigonometric ratios and solve problems involving right triangles. (CCSS: G-SRT)
   i. Explain that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (CCSS: G-SRT.6)
   ii. Explain and use the relationship between the sine and cosine of complementary angles. (CCSS: G-SRT.7)
   iii. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ (CCSS: G-SRT.8)

d. Prove and apply trigonometric identities. (CCSS: F-TF)
   i. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$. (CCSS: F-TF.8)
   ii. Use the Pythagorean identity to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. (CCSS: F-TF.8)

e. Understand and apply theorems about circles. (CCSS: G-C)
   i. Identify and describe relationships among inscribed angles, radii, and chords.10 (CCSS: G-C.2)
   ii. Construct the inscribed and circumscribed circles of a triangle. (CCSS: G-C.3)
   iii. Prove properties of angles for a quadrilateral inscribed in a circle. (CCSS: G-C.3)

f. Find arc lengths and areas of sectors of circles. (CCSS: G-C)
   i. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality. (CCSS: G-C.5)
   ii. Derive the formula for the area of a sector. (CCSS: G-C.5)

**Indicates a part of the standard connected to the mathematical practice of Modeling*

21st Century Skills and Readiness Competencies

Inquiry Questions:

1. How can you determine the measure of something that you cannot measure physically?
2. How is a corner square made?
3. How are mathematical triangles different from triangles in the physical world? How are they the same?
4. Do perfect circles naturally occur in the physical world?

Relevance and Application:

1. Analyzing geometric models helps one understand complex physical systems. For example, modeling Earth as a sphere allows us to calculate measures such as diameter, circumference, and surface area. We can also model the solar system, galaxies, molecules, atoms, and subatomic particles.

Nature of Mathematics:

1. Geometry involves the generalization of ideas. Geometers seek to understand and describe what is true about all cases related to geometric phenomena.
2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
3. Mathematicians attend to precision. (MP)

Extended Evidence Outcomes	Extended Readiness Competencies
With appropriate supports, students can: I. Classify regular polygons (no more than 5 sides) according to their similarities. II. Describe the properties of right triangles. III. Demonstrate the similarities of equilateral triangles and squares using 2-D shapes. IV. Identify right angles in the environment. V. Compare angles and side lengths in a triangle. VI. Demonstrate the longest chord of the circle is the diameter (center point of circle provided).	**Content based access skills:** 1. Attaching meaning to mathematical symbols related to polygons 2. Applying technology to solve mathematical equations 3. Manipulating mathematical materials and equipment related to circles and triangles

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: High School

Concepts and skills students master:

3. Objects in the plane can be described and analyzed algebraically

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Express Geometric Properties with Equations. (CCSS: G-GPE) i. Translate between the geometric description and the equation for a conic section. (CCSS: G-GPE) 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem. (CCSS: G-GPE.1) 2. Complete the square to find the center and radius of a circle given by an equation. (CCSS: G-GPE.1) 3. Derive the equation of a parabola given a focus and directrix. (CCSS: G-GPE.2) ii. Use coordinates to prove simple geometric theorems algebraically. (CCSS: G-GPE) 1. Use coordinates to prove simple geometric theorems11 algebraically. (CCSS: G-GPE.4) 2. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.12 (CCSS: G-GPE.5) 3. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (CCSS: G-GPE.6) 4. Use coordinates and the distance formula to compute perimeters of polygons and areas of triangles and rectangles.★ (CCSS: G-GPE.7) **Indicates a part of the standard connected to the mathematical practice of Modeling*	**Inquiry Questions:** 1. What does it mean for two lines to be parallel? 2. What happens to the coordinates of the vertices of shapes when different transformations are applied in the plane? **Relevance and Application:** 1. Knowledge of right triangle trigonometry allows modeling and application of angle and distance relationships such as surveying land boundaries, shadow problems, angles in a truss, and the design of structures. **Nature of Mathematics:** 1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Explore conic sections. II. Explore the possible areas of a rectangle with a fixed perimeter.	**Content based access skills:** 1. Accessing and using communication system to respond to mathematical problems 2. Manipulating mathematical materials and equipment related to cones, conic sections and perimeter

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: High School

Concepts and skills students master:

4. Attributes of two- and three-dimensional objects are measurable and can be quantified

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Explain volume formulas and use them to solve problems. (CCSS: G-GMD) i. Give an informal argument13 for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. (CCSS: G-GMD.1) ii. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ (CCSS: G-GMD.3) b. Visualize relationships between two-dimensional and three-dimensional objects. (CCSS: G-GMD) i. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (CCSS: G-GMD.4) **Indicates a part of the standard connected to the mathematical practice of Modeling*	**Inquiry Questions:** 1. How might surface area and volume be used to explain biological differences in animals? 2. How is the area of an irregular shape measured? 3. How can surface area be minimized while maximizing volume? **Relevance and Application:** 1. Understanding areas and volume enables design and building. For example, a container that maximizes volume and minimizes surface area will reduce costs and increase efficiency. Understanding area helps to decorate a room, or create a blueprint for a new building. **Nature of Mathematics:** 1. Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 4. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Explore the relationship between the volume of a cylinder and a cone or a rectangular prism and a pyramid. II. Explore the two dimensional nets that correspond to three dimensional solids. III. Identify the shapes that correspond to the faces of three dimensional solids.	**Content based access skills:** 1. Attaching meaning to mathematical geometric symbols 2. Manipulating mathematical materials and equipment related to two dimensional and three dimensional shapes

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: High School

Concepts and skills students master:

5. Objects in the real world can be modeled using geometric concepts

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Apply geometric concepts in modeling situations. (CCSS: G-MG) i. Use geometric shapes, their measures, and their properties to describe objects.14★ (CCSS: G-MG.1) ii. Apply concepts of density based on area and volume in modeling situations.15★ (CCSS: G-MG.2) iii. Apply geometric methods to solve design problems.16★ (CCSS: G-MG.3) **Indicates a part of the standard connected to the mathematical practice of Modeling*	**Inquiry Questions:** 1. How are mathematical objects different from the physical objects they model? 2. What makes a good geometric model of a physical object or situation? 3. How are mathematical triangles different from built triangles in the physical world? How are they the same? **Relevance and Application:** 1. Geometry is used to create simplified models of complex physical systems. Analyzing the model helps to understand the system and is used for such applications as creating a floor plan for a house, or creating a schematic diagram for an electrical system. **Nature of Mathematics:** 1. Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians reason abstractly and quantitatively. (MP) 4. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Estimate the area of an irregular shape use manipulatives/tools.	**Content based access skills:** 1. Expressing an understanding the concept of guessing 2. Selecting appropriate technology to solve mathematical problems 3. Manipulating mathematical materials and equipment

[1] e.g., transparencies and geometry software. (CCSS: G-CO.2)
[2] e.g., translation versus horizontal stretch. (CCSS: G-CO.2)
[3] e.g., graph paper, tracing paper, or geometry software. (CCSS: G-CO.5)
[4] Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (CCSS: G-CO.9)
[5] Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (CCSS: G-CO.10)
[6] Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (CCSS: G-CO.11)
[7] Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (CCSS: G-CO.12)
[8] compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc. (CCSS: G-CO.12)
[9] Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (CCSS: G-SRT.4)
[10] Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (CCSS: G-C.2)
[11] For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2). (CCSS: G-GPE.4)
[12] e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point. (CCSS: G-GPE.5)
[13] Use dissection arguments, Cavalieri's principle, and informal limit arguments. (CCSS: G-GMD.1)
[14] e.g., modeling a tree trunk or a human torso as a cylinder. (CCSS: G-MG.1)
[15] e.g., persons per square mile, BTUs per cubic foot. (CCSS: G-MG.2)
[16] e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios. (CCSS: G-MG.3)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Transformations of objects can be used to define the concepts of congruence and similarity

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Verify experimentally the properties of rotations, reflections, and translations.1 (CCSS: 8.G.1) b. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (CCSS: 8.G.3) c. Demonstrate that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. (CCSS: 8.G.2) d. Given two congruent figures, describe a sequence of transformations that exhibits the congruence between them. (CCSS: 8.G.2) e. Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. (CCSS: 8.G.4) f. Given two similar two-dimensional figures, describe a sequence of transformations that exhibits the similarity between them. (CCSS: 8.G.4) g. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.2 (CCSS: 8.G.5)	**Inquiry Questions:** 1. What advantage, if any, is there to using the Cartesian coordinate system to analyze the properties of shapes? 2. How can you physically verify that two lines are really parallel? **Relevance and Application:** 1. Dilations are used to enlarge or shrink pictures. 2. Rigid motions can be used to make new patterns for clothing or architectural design. **Nature of Mathematics:** 1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Compare the similarities and differences between squares and rhombuses. II. Identify the reflection or rotation of a shape.	**Content based access skills:** 1. Attaching meaning to mathematical symbols related to squares and rhombuses 2. Accessing and using a communication system to respond to mathematical questions related to reflection and rotation 3. Manipulating mathematical materials and equipment

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

2. Direct and indirect measurement can be used to describe and make comparisons

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Explain a proof of the Pythagorean Theorem and its converse. (CCSS: 8.G.6) b. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (CCSS: 8.G.7) c. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (CCSS: 8.G.8) d. State the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (CCSS: 8.G.9)	**Inquiry Questions:** 1. Why does the Pythagorean Theorem only apply to right triangles? 2. How can the Pythagorean Theorem be used for indirect measurement? 3. How are the distance formula and the Pythagorean theorem the same? Different? 4. How are the volume formulas for cones, cylinders, prisms and pyramids interrelated? 5. How is volume of an irregular figure measured? 6. How can cubic units be used to measure volume for curved surfaces? **Relevance and Application:** 1. The understanding of indirect measurement strategies allows measurement of features in the immediate environment such as playground structures, flagpoles, and buildings. 2. Knowledge of how to use right triangles and the Pythagorean Theorem enables design and construction of such structures as a properly pitched roof, handicap ramps to meet code, structurally stable bridges, and roads. 3. The ability to find volume helps to answer important questions such as how to minimize waste by redesigning packaging or maximizing volume by using a circular base. **Nature of Mathematics:** 1. Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden. 2. Geometric objects are abstracted and simplified versions of physical objects 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify the horizontal and vertical locations of a point in the coordinate plane (1st quadrant). II. Identify a landmark in the coordinate plane (1st quadrant) by its horizontal and vertical location. III. Compare the capacity of common containers using common qualitative words/phrases (e.g. fit/doesn't fit, more/less).	**Content based access skills:** 1. Attaching meaning to mathematical symbols for locations in the 1st quadrant of the coordinate plane 2. Following directions for mathematical activities 3. Manipulating mathematical materials and equipment in mathematical activities involving capacity

Standard: 4. Shape, Dimension, and Geometric Relationships
Eighth Grade

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[1] Lines are taken to lines, and line segments to line segments of the same length. (CCSS: 8.G.1a)
Angles are taken to angles of the same measure. (CCSS: 8.G.1b)
Parallel lines are taken to parallel lines. (CCSS: 8.G.1c)
[2] For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. (CCSS: 8.G.5)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Modeling geometric figures and relationships leads to informal spatial reasoning and proof

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Draw construct, and describe geometrical figures and describe the relationships between them. (CCSS: 7.G) i. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (CCSS: 7.G.1) ii. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. (CCSS: 7.G.2) iii. Construct triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (CCSS: 7.G.2) iv. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. (CCSS: 7.G.3)	**Inquiry Questions:** 1. Is there a geometric figure for any given set of attributes? 2. How does scale factor affect length, perimeter, angle measure, area and volume? 3. How do you know when a proportional relationship exists? **Relevance and Application:** 1. The understanding of basic geometric relationships helps to use geometry to construct useful models of physical situations such as blueprints for construction, or maps for geography. 2. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures. **Nature of Mathematics:** 1. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning. 2. The relationship between geometric figures can be modeled 3. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings. 4. Mathematicians use appropriate tools strategically. (MP) 5. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Find the perimeter of a square, triangle, and rectangle using manipulatives/tools (whole unit side lengths with a total perimeter no longer than 30 units).	**Content based access skills:** 1. Accessing and using communication system to respond to mathematical problems involving perimeter 2. Selecting appropriate technology to solve mathematical equations

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

2. Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. State the formulas for the area and circumference of a circle and use them to solve problems. (CCSS: 7.G.4) b. Give an informal derivation of the relationship between the circumference and area of a circle. (CCSS: 7.G.4) c. Use properties of supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (CCSS: 7.G.5) d. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (CCSS: 7.G.6)	**Inquiry Questions:** 1. How can geometric relationships among lines and angles be generalized, described, and quantified? 2. How do line relationships affect angle relationships? 3. Can two shapes have the same volume but different surface areas? Why? 4. Can two shapes have the same surface area but different volumes? Why? 5. How are surface area and volume like and unlike each other? 6. What do surface area and volume tell about an object? 7. How are one-, two-, and three-dimensional units of measure related? 8. Why is pi an important number? **Relevance and Application:** 1. The ability to find volume and surface area helps to answer important questions such as how to minimize waste by redesigning packaging, or understanding how the shape of a room affects its energy use. **Nature of Mathematics:** 1. Geometric objects are abstracted and simplified versions of physical objects. 2. Geometers describe what is true about all cases by studying the most basic and essential aspects of objects and relationships between objects. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Determine the distance around a circular item found in the real world (e.g. size of waist, neck, head etc.) to the nearest whole unit using manipulatives/tools (string, measuring tape etc.)	**Content based access skills:** 1. Working cooperatively with others during mathematical activities related to circumference 2. Manipulating mathematical materials and equipment related to measuring circumference

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Objects in space and their parts and attributes can be measured and analyzed

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can a. Develop and apply formulas and procedures for area of plane figures i. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. (CCSS: 6.G.1) ii. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.1) b. Develop and apply formulas and procedures for volume of regular prisms. i. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths. (CCSS: 6.G.2) ii. Show that volume is the same as multiplying the edge lengths of a rectangular prism. (CCSS: 6.G.2) iii. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (CCSS: 6.G.2) c. Draw polygons in the coordinate plan to solve real-world and mathematical problems. (CCSS: 6.G.3) i. Draw polygons in the coordinate plane given coordinates for the vertices. ii. Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. (CCSS: 6.G.3) d. Develop and apply formulas and procedures for the surface area. i. Represent three-dimensional figures using nets made up of rectangles and triangles. (CCSS: 6.G.4) ii. Use nets to find the surface area of figures. (CCSS: 6.G.4) iii. Apply techniques for finding surface area in the context of solving real-world and mathematical problems. (CCSS: 6.G.4)	**Inquiry Questions:** 1. Can two shapes have the same volume but different surface areas? Why? 2. Can two figures have the same surface area but different volumes? Why? 3. What does area tell you about a figure? 4. What properties affect the area of figures? **Relevance and Application:** 1. Knowledge of how to find the areas of different shapes helps do projects in the home and community. For example how to use the correct amount of fertilizer in a garden, buy the correct amount of paint, or buy the right amount of material for a construction project. 2. The application of area measurement of different shapes aids with everyday tasks such as buying carpeting, determining watershed by a center pivot irrigation system, finding the number of gallons of paint needed to paint a room, decomposing a floor plan, or designing landscapes. **Nature of Mathematics:** 1. Mathematicians realize that measurement always involves a certain degree of error. 2. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians reason abstractly and quantitatively. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify obtuse, right and acute angles. II. Determine the area of squares and rectangles using unit squares. III. Measure the length of a line to the nearest inch using a ruler.	**Content based access skills:** 1. Attaching meaning to mathematical symbols related to angles 2. Applying technology to solve mathematical equations 3. Manipulating mathematical materials and equipment related to angles and length

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. Properties of multiplication and addition provide the foundation for volume an attribute of solids.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Model and justify the formula for volume of rectangular prisms. (CCSS: 5.MD.5b) i. Model the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes.1 (CCSS: 5.MD.5b) ii. Show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. (CCSS: 5.MD.5a) iii. Represent threefold whole-number products as volumes to represent the associative property of multiplication. (CCSS: 5.MD.5a) b. Find volume of rectangular prisms using a variety of methods and use these techniques to solve real world and mathematical problems. (CCSS: 5.MD.5a) i. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (CCSS: 5.MD.4) ii. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths. (CCSS: 5.MD.5b) iii. Use the additive nature of volume to find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts. (CCSS: 5.MD.5c)	**Inquiry Questions:** 1. Why do you think a unit cube is used to measure volume? **Relevance and Application:** 1. The ability to find volume helps to answer important questions such as which container holds more. **Nature of Mathematics:** 1. Mathematicians create visual and physical representations of problems and ideas that reveal relationships and meaning. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify appropriate tools to measure volume. II. Describe volume as empty, full, or part full.	**Content based access skills:** 1. Working cooperatively with others during mathematical activities 2. Expressing an understanding that money has a value and can be exchanged for goods and services 3. Manipulating mathematical materials and equipment related to volume 4. Accessing and using communication system to respond to mathematical problems related to volume

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

2. Geometric figures can be described by their attributes and specific locations in the plane

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Graph points on the coordinate plane2 to solve real-world and mathematical problems. (CCSS: 5.G) b. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (CCSS: 5.G.2) c. Classify two-dimensional figures into categories based on their properties. (CCSS: 5.G) i. Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.3 (CCSS: 5.G.3) ii. Classify two-dimensional figures in a hierarchy based on properties. (CCSS: 5.G.4)	**Inquiry Questions:** 1. How does using a coordinate grid help us solve real world problems? 2. What are the ways to compare and classify geometric figures? 3. Why do we classify shapes? **Relevance and Application:** 1. The coordinate grid is a basic example of a system for mapping relative locations of objects. It provides a basis for understanding latitude and longitude, GPS coordinates, and all kinds of geographic maps. 2. Symmetry is used to analyze features of complex systems and to create worlds of art. For example symmetry is found in living organisms, the art of MC Escher, and the design of tile patterns, and wallpaper. **Nature of Mathematics:** 1. Geometry's attributes give the mind the right tools to consider the world around us. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify common three dimensional shapes: cube, sphere, cone, cylinder, and pyramid. II. Identify the appropriate tool (ruler, yardstick and tape measure) you would use to measure lengths (worm vs. hallway). III. Use the line of symmetry to identify multiple representations of a half.	**Content based access skills:** 1. Attaching meaning to mathematical symbols related to three dimensional shapes 2. Selecting appropriate tools to solve mathematical problems

Standard: 4. Shape, Dimension, and Geometric Relationships
Fifth Grade

[1] A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. (CCSS: 5.MD.3a)
A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. (CCSS: 5.MD.3b)
[2] Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. (CCSS: 5.G.1)
Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). (CCSS: 5.G.1)
[3] For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. (CCSS: 5.G.3)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (CCSS: 4.MD) i. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. (CCSS: 4.MD.1) ii. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.1 (CCSS: 4.MD.1) iii. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. (CCSS: 4.MD.2) iv. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS: 4.MD.2) v. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.2 (CCSS: 4.MD.3) b. Use concepts of angle and measure angles. (CCSS: 4.MD) i. Describe angles as geometric shapes that are formed wherever two rays share a common endpoint, and explain concepts of angle measurement.3 (CCSS: 4.MD.5) ii. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (CCSS: 4.MD.6) iii. Demonstrate that angle measure as additive.4 (CCSS: 4.MD.7) iv. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems.5 (CCSS: 4.MD.7)	**Inquiry Questions:** 1. How do you decide when close is close enough? 2. How can you describe the size of geometric figures? **Relevance and Application:** 1. Accurate use of measurement tools allows people to create and design projects around the home or in the community such as flower beds for a garden, fencing for the yard, wallpaper for a room, or a frame for a picture. **Nature of Mathematics:** 1. People use measurement systems to specify the attributes of objects with enough precision to allow collaboration in production and trade. 2. Mathematicians make sense of problems and persevere in solving them. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians attend to precision. (MP)

Extended Evidence Outcomes	Extended Readiness Competencies
With appropriate supports, students can: I. Demonstrate that whole units can be broken into smaller units (seven days in a week, twelve months in a year). II. Indicate the number of angles (corners) in a regular polygon.	**Content based access skills:** 1. Attaching meaning to mathematical symbols related to units 2. Manipulating mathematical materials and equipment related to polygons

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

2. Geometric figures in the plane and in space are described and analyzed by their attributes

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. (CCSS: 4.G.1) b. Identify points, line segments, angles, and perpendicular and parallel lines in two-dimensional figures. (CCSS: 4.G.1) c. Classify and identify two-dimensional figures according to attributes of line relationships or angle size.6 (CCSS: 4.G.2) d. Identify a line of symmetry for a two-dimensional figure.7 (CCSS: 4.G.3)	**Inquiry Questions:** 1. How do geometric relationships help us solve problems? 2. Is a square still a square if it's tilted on its side? 3. How are three-dimensional shapes different from two-dimensional shapes? 4. What would life be like in a two-dimensional world? 5. Why is it helpful to classify things like angles or shapes? **Relevance and Application:** 1. The understanding and use of spatial relationships helps to predict the result of motions such as how articles can be laid out in a newspaper, what a room will look like if the furniture is rearranged, or knowing whether a door can still be opened if a refrigerator is repositioned. 2. The application of spatial relationships of parallel and perpendicular lines aid in creation and building. For example, hanging a picture to be level, building windows that are square, or sewing a straight seam. **Nature of Mathematics:** 1. Geometry is a system that can be used to model the world around us or to model imaginary worlds. 2. Mathematicians look for and make use of structure. (MP) 3. Mathematicians look for and express regularity in repeated reasoning. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Distinguish between parallel and intersecting lines. II. Recognize a line of symmetry in a simple shape. III. Use two shapes to create a new shape. IV. Discriminate between different attributes of shapes (sides, curves, angles).	**Content based access skills:** 1. Working cooperatively with others during mathematical activities 2. Manipulating mathematical materials and equipment related to shapes 3. Accessing and using communication system to respond to mathematical problems related attributes

Standard: 4. Shape, Dimension, and Geometric Relationships
Fourth Grade

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[1] For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ... (CCSS: 4.MD.1)
[2] For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. (CCSS: 4.MD.3)
[3] An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. (CCSS: 4.MD.5a)
An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. (CCSS: 4.MD.5b)
[4] When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. (CCSS: 4.MD.7)
[5] e.g., by using an equation with a symbol for the unknown angle measure. (CCSS: 4.MD.7)
[6] Based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (CCSS: 4.G.2)
[7] as a line across the figure such that the figure can be folded along the line into matching parts. (CCSS: 4.G.3)
Identify line-symmetric figures and draw lines of symmetry. (CCSS: 4.G.3)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. Geometric figures are described by their attributes

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Reason with shapes and their attributes. (CCSS: 3.G) i. Explain that shapes in different categories1 may share attributes2 and that the shared attributes can define a larger category.3 (CCSS: 3.G.1) 1. Identify rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (CCSS: 3.G.1) ii. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.4 (CCSS: 3.G.2)	**Inquiry Questions:** 1. What words in geometry are also used in daily life? 2. Why can different geometric terms be used to name the same shape? **Relevance and Application:** 1. Recognition of geometric shapes allows people to describe and change their surroundings such as creating a work of art using geometric shapes, or design a pattern to decorate. **Nature of Mathematics:** 1. Mathematicians use clear definitions in discussions with others and in their own reasoning. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify common two dimensional shapes: square, circle, triangle, rectangle, and ellipse. II. Identify common three dimensional shapes: cube, sphere, cone, cylinder, rectangular prism. III. Identify a shape as being the same shape in different orientations (square, rectangle or triangle). IV. Identify a shape within a picture (circle, square, triangle, and rectangle). V. Discriminate between different attributes of shapes (sides or curves).	**Content based access skills:** 1. Expressing an understanding of attributes related to geometric shapes 2. Attaching meaning to geometric representations of shapes 3. Manipulating mathematical materials and equipment related to volume

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Third Grade

Concepts and skills students master:

2. Linear and area measurement are fundamentally different and require different units of measure

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Use concepts of area and relate area to multiplication and to addition. (CCSS: 3.MD) i. Recognize area as an attribute of plane figures and apply concepts of area measurement.5 (CCSS: 3.MD.5) ii. Find area of rectangles with whole number side lengths using a variety of methods6 (CCSS: 3.MD.7a) iii. Relate area to the operations of multiplication and addition and recognize area as additive.7 (CSSS: 3.MD.7) b. Describe perimeter as an attribute of plane figures and distinguish between linear and area measures. (CCSS: 3.MD) c. Solve real world and mathematical problems involving perimeters of polygons. (CCSS: 3.MD.8) i. Find the perimeter given the side lengths. (CCSS: 3.MD.8) ii. Find an unknown side length given the perimeter. (CCSS: 3.MD.8) iii. Find rectangles with the same perimeter and different areas or with the same area and different perimeters. (CCSS: 3.MD.8)	**Inquiry Questions:** 1. What kinds of questions can be answered by measuring? 2. What are the ways to describe the size of an object or shape? 3. How does what we measure influence how we measure? 4. What would the world be like without a common system of measurement? **Relevance and Application:** 1. The use of measurement tools allows people to gather, organize, and share data with others such as sharing results from science experiments, or showing the growth rates of different types of seeds. 2. A measurement system allows people to collaborate on building projects, mass produce goods, make replacement parts for things that break, and trade goods. **Nature of Mathematics:** 1. Mathematicians use tools and techniques to accurately determine measurement. 2. People use measurement systems to specify attributes of objects with enough precision to allow collaboration in production and trade. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Construct arrays of one by 2, 3, 4, or 5 unit squares. II. Demonstrate linear measures have a beginning and end point.	**Content based access skills:** 1. Selecting appropriate technology to solve mathematical equations 2. Attaching meaning to mathematical symbols

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Third Grade

Concepts and skills students master:

3. Time and attributes of objects can be measured with appropriate tools

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (CCSS: 3.MD) i. Tell and write time to the nearest minute. (CCSS: 3.MD.1) ii. Measure time intervals in minutes. (CCSS: 3.MD.1) iii. Solve word problems involving addition and subtraction of time intervals in minutes8 using a number line diagram. (CCSS: 3.MD.1) iv. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (CCSS: 3.MD.2) v. Use models to add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units.9 (CCSS: 3.MD.2)	**Inquiry Questions:** 1. Why do we need standard units of measure? 2. Why do we measure time? **Relevance and Application:** 1. A measurement system allows people to collaborate on building projects, mass produce goods, make replacement parts for things that break, and trade goods. **Nature of Mathematics:** 1. People use measurement systems to specify the attributes of objects with enough precision to allow collaboration in production and trade. 2. Mathematicians use appropriate tools strategically. (MP) 3. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Tell time to the half hour using a digital or analog clock. II. Measure an object to the nearest whole unit using a ruler (up to 12 inches).	**Content based access skills:** 1. Attaching meaning to mathematical symbols 2. Manipulating mathematical materials used for measurement

[1] e.g., rhombuses, rectangles, and others. (CCSS: 3.G.1)
[2] e.g., having four sides. (CCSS: 3.G.1)
[3] e.g., quadrilaterals. (CCSS: 3.G.1)
[4] For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape. (CCSS: 3.G.2)
[5] A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (CCSS: 3.MD.5a)
A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. (CCSS: 3.MD.5b)
[6] A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (CCSS: 3.MD.5a)
A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. (CCSS: 3.MD.5b)
Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). (CCSS: 3.MD.6)
Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (CCSS: 3.MD.7a)
Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (CCSS: 3.MD.7b)
[7] Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. (CCSS: 3.MD.7d)
Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. (CCSS: 3.MD.7c)
[8] e.g., by representing the problem on a number line diagram. (CCSS: 3.MD.1)
[9] e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (CCSS: 3.MD.2)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Second Grade

Concepts and skills students master:

1. Shapes can be described by their attributes and used to represent part/whole relationships

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (CCSS: 2.G.1) b. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (CCSS: 2.G.1) c. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (CCSS: 2.G.2) d. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. (CCSS: 2.G.3) e. Recognize that equal shares of identical wholes need not have the same shape. (CCSS: 2.G.3)	**Inquiry Questions:** 1. How can we describe geometric figures? 2. Is a half always the same size and shape? **Relevance and Application:** 1. Fairness in sharing depends on equal quantities, such as sharing a piece of cake, candy bar, or payment for a chore. 2. Shapes are used to communicate how people view their environment. 3. Geometry provides a system to describe, organize, and represent the world around us. **Nature of Mathematics:** 1. Geometers use shapes to describe and understand the world. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians model with mathematics. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify common two dimensional shapes: square, circle, triangle, and rectangle. II. Identify common three dimensional shapes: cube, sphere, cone, and cylinder.	**Content based access skills:** 1. Manipulating geometric materials 2. Accessing and using communication system to respond to mathematical questions regarding two and three dimensional shapes

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Second Grade

Concepts and skills students master:

2. Some attributes of objects are measurable and can be quantified using different tools

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Measure and estimate lengths in standard units. (CCSS: 2.MD) i. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. (CCSS: 2.MD.1) ii. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (CCSS: 2.MD.2) iii. Estimate lengths using units of inches, feet, centimeters, and meters. (CCSS: 2.MD.3) iv. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (CCSS: 2.MD.4) b. Relate addition and subtraction to length. (CCSS: 2.MD) i. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units[1] and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.MD.5) ii. Represent whole numbers as lengths from 0 on a number line[2] diagram and represent whole-number sums and differences within 100 on a number line diagram. (CCSS: 2.MD.6) c. Solve problems time and money. (CCSS: 2.MD) i. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (CCSS: 2.MD.7) ii. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.[3] (CCSS: 2.MD.8)	**Inquiry Questions:** 1. What are the different things we can measure? 2. How do we decide which tool to use to measure something? 3. What would happen if everyone created and used their own rulers? **Relevance and Application:** 1. Measurement is used to understand and describe the world including sports, construction, and explaining the environment. **Nature of Mathematics:** 1. Mathematicians use measurable attributes to describe countless objects with only a few words. 2. Mathematicians use appropriate tools strategically. (MP) 3. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify standard tools associated with measurement (clock, ruler, measuring cup, scale). II. Measure common objects with non-standard units (e.g. hands, paper clips, etc) up to 12 units. III. Compare lengths of objects and identify as longer/shorter IV. Identify coins (pennies, nickels). (PFL) V. Recognize coins and bills can be exchanged for goods, merchandise and/or services. (PFL) VI. Tell time to the hour using digital and analog clocks.	**Content based access skills:** 1. Manipulating mathematical materials and equipment used for measurement 2. Attaching meaning to symbols for money 3. Demonstrating and understanding of time and time management

Standard: 4. Shape, Dimension, and Geometric Relationships Second Grade

[1] e.g., by using drawings (such as drawings of rulers). (CCSS: 2.MD.5)
[2] with equally spaced points corresponding to the numbers 0, 1, 2, ... (CCSS: 2.MD.6)
[3] Example: If you have 2 dimes and 3 pennies, how many cents do you have? (CCSS: 2.MD.6)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: First Grade

Concepts and skills students master:

1. Shapes can be described by defining attributes and created by composing and decomposing

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Distinguish between defining attributes1 versus non-defining attributes.2 (CCSS: 1.G.1) b. Build and draw shapes to possess defining attributes. (CCSS: 1.G.1) c. Compose two-dimensional shapes3 or three-dimensional shapes4 to create a composite shape, and compose new shapes from the composite shape. (CCSS: 1.G.2) d. Partition circles and rectangles into two and four equal shares. (CCSS: 1.G.3) i. Describe shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. (CCSS: 1.G.3) ii. Describe the whole as two of, or four of the equal shares.5 (CCSS: 1.G.3)	**Inquiry Questions:** 1. What shapes can be combined to create a square? 2. What shapes can be combined to create a circle? **Relevance and Application:** 1. Many objects in the world can be described using geometric shapes and relationships such as architecture, objects in your home, and things in the natural world. Geometry gives us the language to describe these objects. 2. Representation of ideas through drawing is an important form of communication. Some ideas are easier to communicate through pictures than through words such as the idea of a circle, or an idea for the design of a couch. **Nature of Mathematics:** 1. Geometers use shapes to represent the similarity and difference of objects. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify common two dimensional shapes: square, circle, triangle (obtuse, acute, right, equilateral, and isosceles). II. Identify common three dimensional shapes: cube, sphere, and cone. III. Sort a set of two differently shaped objects (circle, square, and similar triangle). IV. Match a shape to a real object or picture of a real object (square, circle, triangle, sphere, cube, and cone).	**Content based access skills:** 1. Manipulating geometric materials 2. Attaching meaning to symbols related to two and three dimensional objects 3. Expressing personal preferences and choices related to shapes

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: First Grade

Concepts and skills students master:

2. Measurement is used to compare and order objects and events

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Measure lengths indirectly and by iterating length units. (CCSS: 1.MD) i. Order three objects by length; compare the lengths of two objects indirectly by using a third object. (CCSS: 1.MD.1) ii. Express the length of an object as a whole number of length units.6 (CCSS: 1.MD.2) b. Tell and write time. (CCSS: 1.MD) i. Tell and write time in hours and half-hours using analog and digital clocks. (CCSS: 1.MD.3)	**Inquiry Questions:** 1. How can you tell when one thing is bigger than another? 2. Why do we measure objects and time? 3. How are length and time different? How are they the same? **Relevance and Application:** 1. Time measurement is a means to organize and structure each day and our lives, and to describe tempo in music. 2. Measurement helps to understand and describe the world such as comparing heights of friends, describing how heavy something is, or how much something holds. **Nature of Mathematics:** 1. With only a few words, mathematicians use measurable attributes to describe countless objects. 2. Mathematicians use appropriate tools strategically. (MP) 3. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Sequence three objects by size. II. Use terms to describe order in a sequence (first, next, last). III. Sequence up to three units of time related to the day (morning, afternoon and night or breakfast, lunch and dinner).	**Content based access skills:** 1. Sequencing mathematical objects 2. Expressing an understanding of size 3. Demonstrating and understanding of time

[1] e.g., triangles are closed and three-sided. (CCSS: 1.G.1)
[2] e.g., color, orientation, overall size. (CCSS: 1.G.1)
[3] rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles. (CCSS: 1.G.2)
[4] cubes, right rectangular prisms, right circular cones, and right circular cylinders. (CCSS: 1.G.2)
[5] Understand for these examples that decomposing into more equal shares creates smaller shares. (CCSS: 1.G.3)
[6] By laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. (CCSS: 1.MD.2)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Kindergarten

Concepts and skills students master:

1. Shapes can be described by characteristics and position and created by composing and decomposing

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). (CCSS: K.G) i. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. (CCSS: K.G.1) ii. Correctly name shapes regardless of their orientations or overall size. (CCSS: K.G.2) iii. Identify shapes as two-dimensional1 or three dimensional.2 (CCSS: K.G.3) b. Analyze, compare, create, and compose shapes. (CCSS: K.G) i. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts3 and other attributes.4 (CCSS: K.G.4) ii. Model shapes in the world by building shapes from components5 and drawing shapes. (CCSS: K.G.5) iii. Compose simple shapes to form larger shapes.6 (CCSS: K.G.6)	**Inquiry Questions:** 1. What are the ways to describe where an object is? 2. What are all the things you can think of that are round? What is the same about these things? 3. How are these shapes alike and how are they different? 4. Can you make one shape with other shapes? **Relevance and Application:** 1. Shapes help people describe the world. For example, a box is a cube, the Sun looks like a circle, and the side of a dresser looks like a rectangle. 2. People communicate where things are by their location in space using words like next to, below, or between. **Nature of Mathematics:** 1. Geometry helps discriminate one characteristic from another. 2. Geometry clarifies relationships between and among different objects. 3. Mathematicians model with mathematics. (MP) 4. Mathematicians look for and make use of structure. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Identify two dimensional shapes: circle and square. II. Explore three dimensional shapes: Sphere and cube. III. Match like shapes (2-D/2-D, 3-D/3-D) (circle, square, sphere, cube) IV. Explore shapes in the world by building shapes from components (e.g., sticks and clay balls).	**Content based access skills:** 1. Maintaining attention to shapes 2. Accessing and using communication system to respond to mathematical questions regarding two and three dimensional shapes

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Kindergarten

Concepts and skills students master:

2. Measurement is used to compare and order objects

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Describe and compare measurable attributes. (CCSS: K.MD) i. Describe measurable attributes of objects, such as length or weight. (CCSS: K.MD.1) ii. Describe several measurable attributes of a single object. (CCSS: K.MD.1) iii. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference.7 (CCSS: K.MD.2) iv. Order several objects by length, height, weight, or price (PFL) b. Classify objects and count the number of objects in each category. (CCSS: K.MD) i. Classify objects into given categories. (CCSS: K.MD.3) ii. Count the numbers of objects in each category. (CCSS: K.MD.3) iii. Sort the categories by count. (CCSS: K.MD.3)	**Inquiry Questions:** 1. How can you tell when one thing is bigger than another? 2. How is height different from length? **Relevance and Application:** 1. Measurement helps to understand and describe the world such as in cooking, playing, or pretending. 2. People compare objects to communicate and collaborate with others. For example, we describe items like the long ski, the heavy book, the expensive toy. **Nature of Mathematics:** 1. A system of measurement provides a common language that everyone can use to communicate about objects. 2. Mathematicians use appropriate tools strategically. (MP) 3. Mathematicians attend to precision. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Use big/little, more/less to compare quantities or size. II. Explore a simple pictograph.	**Content based access skills:** 1. Manipulating mathematical materials and equipment 2. Attaching meaning to a mathematical symbol related to size and quantity

[1] lying in a plane, "flat". (CCSS: K.G.3)
[2] "solid". (CCSS: K.G.3)
[3] e.g., number of sides and vertices/"corners". (CCSS: K.G.4)
[4] e.g., having sides of equal length. (CCSS: K.G.4)
[5] e.g., sticks and clay balls. (CCSS: K.G.5)
[6] For example, "Can you join these two triangles with full sides touching to make a rectangle?" (CCSS: K.G.6)
[7] For example, directly compare the heights of two children and describe one child as taller/shorter. (CCSS: K.MD.2)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Preschool

Concepts and skills students master:

1. Shapes can be observed in the world and described in relation to one another

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: 1. Match, sort, group and name basic shapes found in the natural environment 2. Sort similar groups of objects into simple categories based on attributes 3. Use words to describe attributes of objects 4. Follow directions to arrange, order, or position objects	**Inquiry Questions:** 1. How do we describe where something is? 2. Where do you see shapes around you? 3. How can we arrange these shapes? 4. Why do we put things in a group? 5. What is the same about these objects and what is different? 6. What are the ways to sort objects?
	Relevance and Application: 1. Shapes and position help students describe and understand the environment such as in cleaning up, or organizing and arranging their space. 2. Comprehension of order and position helps students learn to follow directions. 3. Technology games can be used to arrange and position objects. 4. Sorting and grouping allows people to organize their world. For example, we set up time for clean up, and play.
	Nature of Mathematics: 1. Geometry affords the predisposition to explore and experiment. 2. Mathematicians organize objects in different ways to learn about the objects and a group of objects. 3. Mathematicians attend to precision. (MP) 4. Mathematicians look for and make use of structure. (MP)

Extended Evidence Outcomes	Extended Readiness Competencies
With appropriate supports, students can: I. Sort simple objects based on one attribute.	**Content based access skills:** 1. Expressing an understanding of same

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Preschool

Concepts and skills students master:

2. Measurement is used to compare objects

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: 1. Describe the order of common events 2. Group objects according to their size using standard and non-standard forms (height, weight, length, or color brightness) of measurement 3. Sort coins by physical attributes such as color or size (PFL)	**Inquiry Questions:** 1. How do we know how big something is? 2. How do we describe when things happened?
	Applying Mathematics in Society and Using Technology: 1. Understanding the order of events allows people to tell a story or communicate about the events of the day. 2. Measurements helps people communicate about the world. For example, we describe items like big and small cars, short and long lines, or heavy and light boxes.
	Nature of Mathematics: 1. Mathematicians sort and organize to create patterns. Mathematicians look for patterns and regularity. The search for patterns can produce rewarding shortcuts and mathematical insights. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians use appropriate tools strategically. (MP)
Extended Evidence Outcomes	**Extended Readiness Competencies**
With appropriate supports, students can: I. Use big/little to compare size.	**Content based access skills:** 1. Manipulating mathematical materials and equipment 2. Attaching meaning to a mathematical symbol related to size

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